Cosmology in Nonlocal Gravity

非局域引力中的宇宙学

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Contents

目录

参考文献 1492

Abstract

摘要

In this chapter we review the recent developments of realizing R^2 -like inflation in the framework of a most general UV nonlocal extension of Einstein's general theory of relativity (GR). It is a well-motivated robust approach toward quantum

本章我们综述在爱因斯坦广义相对论 (GR) 最一般的紫外非局域推广框架下，实现 R^2 类暴涨的最新进展。这是通往量子

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gravity. In the past decades, nonlocal gravitational theories that are quadratic in curvature have been understood to be ghost-free and super-renormalizable around maximally symmetric spacetimes. However, in the context of early Universe cosmology, we show that one must go beyond the quadratic curvature nonlocal gravity in order to achieve a consistent ghost-free framework of Universe evolution from quasi-de Sitter to Minkowski spacetime. In this regard, we discuss a construction of a most general nonlocal gravity action that leads to R^2 -like inflation and discuss the corresponding observational predictions for the scalar and tensor spectral tilts, tensor-to-scalar ratio, and the primordial non-Gaussianities. We present an analysis of how the nonlocal inflationary cosmology goes beyond the established notions of effective field theories of inflation. Finally, we comment on some open questions and prospects of higher curvature nonlocal gravity on its way of achieving the UV completion.

引力的动机充分的可靠研究方向。过去数十年间，人们已经认识到，曲率二次项的非局域引力理论在最大对称时空背景下是无鬼场且超可重整化的。但我们表明，在早期宇宙宇宙学的语境下，为了得到一个从拟德西特时空演化到闵氏时空的自洽无鬼框架，必须超出二次曲率非局域引力的范畴。据此，我们讨论了可导出 R^2 类暴涨的最一般非局域引力作用量构造，还讨论了该模型对标量和张量谱倾角、张标比以及原初非高斯性的相应观测预言。我们分析了非局域暴涨宇宙学如何突破已有的暴涨有效场论观念。最后，我们对高曲率非局域引力在实现紫外完备过程中存在的若干开放性问题与研究前景进行了评述。

Keywords

关键词

Models of quantum gravity · Nonlocality and inflationary cosmology

量子引力模型 · 非定域性与暴胀宇宙学

Introduction

引言

Most often a prominent goal in the quantum gravity research is to achieve a short distance and time scale modification of Einstein’s general theory of relativity (GR) and to find its ultraviolet (UV) completion via demanding unitarity and renormalizability around Minkowski spacetime. However, when it comes to probing any theory of quantum gravity, one needs to look for the applications in the context of cosmology and black hole physics which implies the need of understanding quantum nature of gravity in curved and dynamical spacetime. This means UV completion of gravity should be restricted not only to Minkowski spacetime but also to the curved spacetime. The goal of obtaining a unitary and renormalizable gravity theory around Minkowski background provides the essential steps we should take to modify GR, while the goal of explaining the early Universe and black hole physics produces practical restrictions on achieving a UV completion and also constrains the parameter space of the UV complete theory. The so far best example of a renormalizable gravity is represented by Stelle's fourth-order action [1]

量子引力研究通常有一个核心目标: 对爱因斯坦广义相对论 (GR) 进行短距离和短时间尺度的修正，并通过要求理论在闵氏时空周围满足幺正性和可重整化性，找到其紫外 (UV) 完备化。然而，若要检验任意量子引力理论，我们需要在宇宙学和黑洞物理的背景下寻找其应用，这意味着我们需要理解弯曲动态时空下引力的量子本质。也就是说，引力的紫外完备化不应仅局限于闵氏时空，还应当适用于弯曲时空。在闵氏背景下得到幺正、可重整化引力理论，是我们修正广义相对论需要迈出的核心步骤；而解释早期宇宙和黑洞物理的目标，则对实现紫外完备化提出了实际限制，同时也约束了紫外完备理论的参数空间。目前可重整化引力最典型的例子是 Stelle 的四阶作用量 [1]

$$S_4 = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + \frac{f_0}{2}R^2 + \frac{f_{0C}}{2}W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}\right], \tag{1}$$

where f_0, f_{0C} are dimensionless coefficients and $M_p = \frac{1}{\sqrt{8\pi G}}$ is the reduced Planck mass. It is a renormalizable theory of gravity which is an extension of GR with additional Ricci scalar square $\left(R^2\right)$ and Weyl tensor square $\left(W^2 = W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}\right)$ in the action. However it is well known that Stelle's gravity is non-unitary due to the presence of an additional tensor ghost. On the other hand, the model of Starobinsky inflation based on a quadratic scalar curvature extension of GR [2],

其中 f_0, f_{0C} 是无量纲系数, $M_p = \frac{1}{\sqrt{8\pi G}}$ 是约化普朗克质量。这是一个可重整化的引力理论, 是广义相对论的拓展, 在作用量中加入了额外的里奇标量平方项 $\left(R^2\right)$ 和外尔张量平方项 $\left(W^2 = W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}\right.$)。但众所周知，Stelle 引力是非幺正的，因为理论中存在一个额外的张量鬼场。另一方面，基于广义相对论二次标量曲率拓展的 Starobinsky 暴胀模型 [2]

$$S^{\rm local}_{\rm R+R^2} = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + \frac{f_0}{2}R^2\right], \tag{2}$$

where $f_0 = \frac{M_p^2}{6M^2}$ with M being a mass of an additional scalar degree of freedom, is strongly supported by the latest observations of cosmic microwave background (CMB) by Planck satellite [3] with the value of $M \approx 1.3 \times 10^{-5} M_p$ for the $N = (50 - 60)$ number of e-foldings before the end of inflation. The success of Starobinsky inflation makes a strong point to the higher curvature modification of gravity, and in the context of Stelle gravity we must impose the inequality $f_0 \gg f_{0C}$ in order to push the ghost away from affecting the primordial physics. Furthermore, the presence of R^2 term as an extension of GR is inevitable if one considers the contribution of one-loop matter corrections to the graviton self-energy [4]. Although we do not know what is a theory of gravity at the Planck scale, we can greatly expect the necessity of higher curvature terms [5-7]. In this sense the Weyl tensor square term is important for quantum gravity [8], although we require $f_0 \gg f_{0C}$ for the stability of inflation (It is interesting to note here that the Weyl ghost can be avoided by using new quantum prescriptions [9] or by considering the dressed propagator of graviton with a notion of unstable ghosts [10]. However, further investigation is needed in the scope of quantum field theory in curved spacetime in order to check if the ghost problem can be fully alleviated.). To achieve a consistent theory of quantum gravity it is essential to go beyond the Stelle gravity, but any finite derivative extension of it would obviously lead to the well-known Ostrogradsky instability [11].

其中 $f_0 = \frac{M_p^2}{6M^2}$ ，M 是额外标量自由度的质量，该模型得到了普朗克卫星对宇宙微波背景 (CMB) 最新观测的强力支持 [3]，暴胀结束前 $N = (50 - 60)$ 个 e-fold 对应的结果满足 $M \approx 1.3 \times 10^{-5} M_p$ 。Starobinsky 暴胀的成功有力地证明了引力高阶曲率修正的合理性，在 Stelle 引力的框架下，我们必须施加不等式 $f_0 \gg f_{0C}$ ，以避免鬼场影响原初物理。此外，如果考虑一圈物质修正对引力子自能的贡献 [4]，那么 R^2 项作为广义相对论的拓展是必然存在的。尽管我们尚不清楚普朗克尺度下的引力理论究竟是什么，但我们完全可以确定高阶曲率项是必然存在的 [5-7]。从这个角度来说，外尔张量平方项对量子引力十分重要 [8]，不过为了保证暴胀的稳定性我们要求 $f_0 \gg f_{0C}$ (这里值得注意的是，可以通过新的量子规则 [9]，或者引入不稳定鬼场的概念重新处理引力子穿衣传播子 [10]，从而规避外尔鬼场问题。但要检验鬼场问题能否被完全解决，还需要在弯曲时空量子场论的框架下开展进一步研究。)。要得到自洽的量子引力理论，必须超越 Stelle 引力，但对它做任何有限导数拓展都会显然地引发众所周知的 Ostrogradsky 不稳定性 [11]。

Several approaches toward quantum gravity do have a common feature known as nonlocality which arises due to an effect of infinite derivative terms in the action [12]. Considering nonlocality as a principle, a straightforward extension of Stelle gravity with an infinite number of covariant derivatives has been shown to resolve the ghost problem of the theory [13-16] and has been projected to be a prominent candidate for a quantum gravity. The nonlocal quadratic curvature gravity (NLQG) theories in particular have been extensively studied in the past decade in the contexts of unitarity, renormalizability with some applications in cosmology and astrophysics [17-24]. The NLQG is shown to be ghost-free around Minkowski spacetime (raising chances for it to be unitary), and renormalizability of the theory is so far shown by the methods of power counting within the restricted class of form factors which are analytic infinite derivative (AID) operators [14, 16, 25, 26] (see also the recent review article [27] and the references therein) (Also, in some restricted class of nonlocal gravity actions, renormalizability in the de Sitter and anti-de Sitter backgrounds also has been achieved [28].). In the recent years, NLQG has been extensively studied in the scope of embedding $R + R^2$ or Starobinsky inflation in a UV complete framework [12,19,29-31]. Proven successful this embedding of R^2 inflation in NLQG has led to a nonlocal R^2 -like inflation which has interesting observational consequences in the form of new predictions for the inflationary observables such as tensor-to-scalar ratio (r) , tensor tilt (n_t) , and primordial non-Gaussianities (PNGs) [12]. Therefore, future CMB and primordial gravitational wave (PGW) observations certainly will be able to shed light on signatures of inflation in nonlocal gravity [12]. As we discussed earlier, cosmological or astrophysical implications of any framework of quantum gravity will further anchor our understanding of quantum gravity. Indeed even though NLQG framework is successfully understood to be a super-renormalizable theory of quantum gravity, the cosmological application of it does demand the further extension of NLQG if we want to achieve a ghost-freeness simultaneously in inflationary and Minkowski backgrounds. This has been shown in the recent developments of generalized nonlocal R^2 -like inflation [32,33]. It has been well known that the ghost-free requirement of NLQG requires nonlocal form factors to be analytic functions not only of the d'Alembertian operator but also of background curvature [28, 34-36]. This implies that the NLQG is an incomplete framework, and it could probably be a part of a more generic model. From the cosmological application point of view, NLQG has been extended to generalized nonlocal quantum gravity (GNLQG) with higher curvature nonlocal terms, so that one can achieve a consistent regime of R^2 -like inflation [32,33]. These developments are significant advances compared to the earlier known NLQG. This in turn extends and sets new goals for our understanding of quantum gravity. In [32, 33] , a thorough study of R^2 -like inflation in GNLQG has been performed, and inflationary observables are derived bringing new insight into the physics of primordial Universe (We note that in the context of a version of NLQG without any propagating scalar degree of freedom, an alternative framework has been proposed recently which predicts $r \gtrsim 0.01$ [37, 38] . In this review, we stick ourselves to the discussion of early Universe inflationary cosmology rather to any of its ad hoc alternatives).

多种量子引力研究方法确实存在一个共同特征，即非定域性，它由作用量中的无限导数项效应产生[12]。将非定域性作为一项基本原理，已有研究表明，对施特勒引力进行直接拓展，引入无限多个协变导数，可以解决该理论的鬼问题 [13-16]，该理论已被视作量子引力的重要候选理论。非局域二次曲率引力 (NLQG) 理论在过去十年中得到了广泛研究，研究涵盖幺正性、可重整性，同时已在宇宙学和天体物理学中得到若干应用 [17-24]。研究表明，NLQG 在闵氏时空周围不存在鬼 (这提高了它满足幺正性的可能性)，目前通过幂次计数方法证明了其可重整性，该结论成立于限制类的形状因子，即解析无限导数 (AID) 算符 [14, 16, 25, 26] (另见近期综述文章 [27] 及其中参考文献)(此外，在某些受限类的非局域引力作用量中，德西特和反德西特背景下的可重整性也已经实现 [28])。近年来，NLQG 在嵌入 $R + R^2$ 或斯塔罗宾斯基暴胀到紫外完备框架的范围内得到了广泛研究 [12,19,29-31]。事实证明，将 R^2 暴胀嵌入 NLQG 取得了成功，由此得到了非局域类 R^2 暴胀，该模型具有值得关注的观测效应，能够对暴胀观测量给出新预言，比如张量-标量比 (r)、张量倾角 (n_t) 以及原初非高斯性 (PNG)[12]。因此，未来的 CMB 和原初引力波 (PGW) 观测一定能够揭示非局域引力中暴胀的特征 [12]。正如我们此前讨论的，任何量子引力框架的宇宙学或天体物理学应用都会进一步加深我们对量子引力的理解。事实上，尽管 NLQG 框架已经被公认为是一套自洽的超可重整化量子引力理论，但如果我们想要同时在暴胀背景和闵氏背景中实现无鬼性，那么将它应用于宇宙学时仍然需要对 NLQG 做进一步拓展。这一点已经在广义非局域类 R^2 暴胀的最新研究进展中得到了证明 [32,33]。众所周知，NLQG 的无鬼要求要求非局域形状因子不仅是达朗贝尔算符的解析函数，也必须是背景曲率的解析函数 [28, 34-36]。这意味着 NLQG 是一个不完备的框架，它可能只是一个更一般模型的一部分。从宇宙学应用的角度来看，NLQG 已经被拓展为含更高曲率非局域项的广义非局域量子引力 (GNLQG)，从而可以得到类 R^2 暴胀的自洽区域 [32,33]。和此前已知的 NLQG 相比，这些进展是重大的进步，进而也为我们理解量子引力拓展了方向、设立了新目标。在 [32, 33] 中，研究者已经对 GNLQG 框架下的类 R^2 暴胀开展了全面研究，推导出了暴胀观测量，为原初宇宙的物理学带来了新的见解 (我们注意到，在一类不包含任何传播标量自由度的 NLQG 语境下，近期已经有人提出了一个替代框架，该框架预言 $r \gtrsim 0.01$ [37, 38] 。在本综述中，我们仅讨论早期宇宙暴胀宇宙学，不讨论任何特定的替代框架)。

Inflationary cosmology so far has been intensively dominated by the numerous frameworks of scalar fields that are motivated from various approaches to UV complete physics such as string theory and supergravity [39-41]. However, already in the pioneer paper [2], it had been demonstrated that a viable and full inflationary model including creation and heating of matter after inflation can be realized purely geometrically without introduction of scalar fields. Especially, after the release of first Planck data [3,42], there has been an increasing activity of finding extensions to the R^2 -like framework combined with UV complete setups [43], and in this regard a possible UV completion of R^2 gravity within the scope of nonlocal gravity not only adds a new perspective to the current wide activities of inflationary cosmology but also highlights the possible role of nonlocality in the early Universe physics. Namely, the recent studies [32, 33] specifically put these efforts on par to the contemporary attempts of realizing of the physics of inflation through the so-called effective field theory (EFT) of a single field inflation or a multifield inflation [44, 45]. To be more precise, the EFT of single-field inflation (EFT-SI) prescribes that any deviation from a single-field inflationary slow-roll consistency relation ($r = -8n_t$) requires the sound speeds of scalar and tensor degrees of freedom to deviate from unity. However, the nonlocal R^2 -like inflation both in NLQG and in GNLQG [19,32] violates the consistency relation $r = -8n_t$ preserving the unit sound speeds of perturbed modes. Moreover, in the nonlocal R^2 -like inflation, we obtain both positive and negative values of tensor tilt which is entirely a new effect in the context of geometric formulations of inflation. Especially, the possibilities of positive values for n_t posit a counterexample to the alternative frameworks of inflation such as string gas cosmology [46]. Furthermore, the studies of primordial non-Gaussianities (PNG) in the nonlocal gravity inflation have revealed that one can obtain nontrivial shapes of PNGs that are otherwise thought to be not possible in standard EFT formulations of a single-field and multifield inflation [19,33]. This is purely due to the nonlocal nature of gravity, and it opens a new window for possible observational outcomes of inflation that should be kept in mind when interpreting results of future CMB and PGW probes such as CMBS4 [47], LiteBIRD [48], CORE [49], and e-LISA [50]. With respect to PNGs, the nonlocal R^2 -like inflation presents a new target to our future CMB and Large-Scale Structure observations reaching $f_{\rm NL} \sim O(1)$ or slightly more [51-56].

迄今为止，暴胀宇宙学的诸多研究框架都以标量场为主导，这类标量场来自弦论、超引力等多种紫外完备物理方法的启发 [39-41]。然而早在开创性文献 [2] 中，研究者就已证明，无需引入标量场，仅依靠纯几何构造就能得到可行的完整暴胀模型，涵盖暴胀后物质的生成与再加热过程。特别是在普朗克卫星首批数据公布后 [3,42]，越来越多研究致力于寻找结合紫外完备框架的 R^2 类框架扩展方向 [43]；就此而言，非局域引力框架下 R^2 引力可能的紫外完备方案不仅为当前暴胀宇宙学的大量研究提供了新视角，也凸显了非局域性在早期宇宙物理中的潜在作用。也就是说，近期研究 [32,33] 明确将这类工作与当代通过单场暴胀或多场暴胀的所谓有效场论 (EFT) 实现暴胀物理的研究放在同等重要的位置 [44,45]。更准确地说，单场暴胀有效场论 (EFT-SI) 指出，任何偏离单场暴胀慢滚一致性关系 ($r = -8n_t$) 的情况，都要求标量和张量自由度的声速偏离 1。但 NLQG 和 GNLQG 中的非局域 R^2 类暴胀 [19,32] 都在保留微扰模声速为 1 的同时，违反了一致性关系 $r = -8n_t$ 。此外，在非局域 R^2 类暴胀中，我们可以得到张量倾角的正值和负值，这在暴胀的几何构造中是一种全新效应。特别是 n_t 取正值的可能性，对弦气宇宙学等其他暴胀框架构成了反例 [46]。更进一步，非局域引力暴胀中对原初非高斯性 (PNG) 的研究表明，这类模型可以得到非平庸的 PNG 形状，而人们通常认为这在标准的单场、多场暴胀 EFT 构造中是不可能实现的 [19,33]。这一性质完全来源于引力的非局域本质，它为暴胀可能的观测结果打开了新窗口，在解读 CMBS4[47]、LiteBIRD[48]、CORE[49]、e-LISA[50] 等未来 CMB 和原初引力波探测的结果时，需要牢记这一点。就原初非高斯性而言，非局域 R^2 类暴胀为未来 CMB 和大尺度结构观测达到 $f_{\rm NL} \sim O(1)$ 甚至更高精度提供了新的研究目标 [51-56]。

We organize the chapter as follows. In section "The Form Factors of Quadratic Curvature Nonlocal Gravity in Flat and Curved Backgrounds" we review the NLQG framework and discuss the ghost-free form factors in maximally symmetric spacetimes (MSSs) [28,30,31,34]. We highlight that the form factors of de Sitter (dS) background do depend on the background curvature. We present the versions of NLQG that are suitable for realizing exactly the R^2 -like inflation. Then we discuss how the ghost-free conditions in dS and quasi-dS (inflationary) background differ slightly and how the inflationary framework does require an extension of NLQG with higher curvature nonlocal terms. This essential extension is called GNLQG which is the result of the recent study [32]. In section "Generalized Nonlocal Gravity and R^2 -Like Inflation" we discuss in detail the construction of GNLQG within the scope of embedding R^2 -like inflation. We also present a brief discussion of UV completion aspects of GNLQG. In section "Predictions of Generalized Nonlocal R^2 -Like Inflation" we review the predictions of GNLQG and how this theory offers a new understanding to the physics of inflation [19, 33]. We discuss in detail the ways one can observationally probe GNLQG with the future cosmological and gravitational wave probes. In section "Lessons from Generalized Nonlocal R^2 -like Inflation and Its Impact on the EFTs of Inflationary Cosmology" we analyze some of the lessons one can learn from the development of GNLQG and discuss the impacts on the popular EFT understanding of inflation. We highlight how one can end up with some features in EFT which cannot be present in a fundamental UV complete framework. In section "Conclusion and Outlook" we conclude with key takeaways from R^2 -like inflation in GNLQG and further future directions which we can explore on our way of reaching a consistent quantum gravity.

我们对本章结构安排如下: 在“平直与弯曲背景下二次曲率非局域引力的形状因子”一节中, 我们回顾了 NLQG 框架，并讨论了最大对称时空 (MSS) 中的无鬼形状因子 [28,30,31,34]。我们强调，德西特 (dS) 背景的形状因子确实依赖于背景曲率。我们介绍了适用于精确实现 R^2 型暴胀的几类 NLQG 版本。随后我们讨论了 dS 背景与准 dS(暴胀) 背景下的无鬼条件为何存在细微差异，以及暴胀框架为何要求对 NLQG 扩展更高阶曲率非局域项。这一必要扩展被称为 GNLQG，是最新研究 [32] 的成果。在“广义非局域引力与 R^2 型暴胀”一节中，我们详细讨论了在嵌入 R^2 型暴胀的框架下 GNLQG 的构造，也简要讨论了 GNLQG 的紫外完备性相关问题。在“广义非局域 R^2 型暴胀的预言”一节中，我们回顾了 GNLQG 的理论预言，以及该理论如何为暴胀物理学提供新的理解 [19, 33]。我们详细讨论了未来宇宙学与引力波观测探测 GNLQG 的可行途径。在“广义非局域 R^2 型暴胀的启示及其对暴胀宇宙学有效场论的影响”一节中，我们分析了 GNLQG 发展带来的启示，讨论了其对流行的暴胀有效场论认知的影响。我们强调了有效场论中为何会出现基础紫外完备框架中不可能存在的一些特征。在“结论与展望”一节中，我们总结了 GNLQG 框架下 R^2 型暴胀的核心要点，并梳理了我们在追寻自洽量子引力道路上可进一步探索的未来方向。

Throughout the chapter we work with a mostly positive metric signature $(-,+,+,+)$, we set $\hbar = c = 1$ and the reduced Planck mass $M_p = \frac{1}{\sqrt{8\pi G}}$. We use overbars to denote background quantities.

本章通篇采用大多数文献使用的正号度规号差 $(-,+,+,+)$ ，我们约定 $\hbar = c = 1$ ，约化普朗克质量为 $M_p = \frac{1}{\sqrt{8\pi G}}$ 。我们用上横线标记背景物理量。

The Form Factors of Quadratic Curvature Nonlocal Gravity in Flat and Curved Backgrounds

平直与弯曲背景下二次曲率非局域引力的形状因子

The action of NLQG which is an analytic infinite derivative (AID) extension of Stelle gravity (1) is given by

本文研究的 NLQG 作用量是施特勒引力 (1) 的解析无穷导数 (AID) 推广，其形式为

$$S_q^{\text{Nonlocal}} = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_p^2 R + R\mathcal{F}_R\left(\Box_s\right)R + W_{\mu\nu\rho\sigma}\mathcal{F}_C\left(\Box_s\right)W^{\mu\nu\rho\sigma}\right],$$

(3)

where $\Box_s = \frac{\Box}{\mathcal{M}_s^2}$ with $\mathcal{M}_s \ll M_p$ being the scale of nonlocality, and the form factors $\mathcal{F}_R\left(\Box_s\right)$ and $\mathcal{F}_C\left(\Box_s\right)$ are the AID operators which can be Taylor expanded as

其中 $\Box_s = \frac{\Box}{\mathcal{M}_s^2}$，$\mathcal{M}_s \ll M_p$ 为非局域性标度，形状因子 $\mathcal{F}_R(\Box_s)$ 与 $\mathcal{F}_C(\Box_s)$ 是可作泰勒展开的 AID 算符，展开形式为

$$\mathcal{F}_R(\Box_s) = \sum_{m=0}^{\infty} f_{mR}\Box_s^m,\ \mathcal{F}_C(\Box_s) = \sum_{n=0}^{\infty} f_{nC}\Box_s^n. \tag{4}$$

At this stage the above gravity theory contains an infinite number of arbitrary parameters given by the infinite set (f_{mR}, f_{mC}) . However, once we demand the theory to be ghost-free, we significantly reduce the freedom of choice for the form factors (4). Then it comes to the question what is the background around which the theory has to be ghost-free and how one can find the structure of the form factors. We can first aim to answer these questions in the simplest background we know which is Minkowski spacetime. We want the theory to be ghost-free around Minkowski not only because it is just the simplest spacetime but also we expect that any curved spacetime must be locally Minkowski. This means that at very short length scales or at very high energy scales (but still much lower than the energy scale of possible breaking of the local Lorentz invariance), we expect the spacetime to be nearly Minkowski. On the other hand, since most of the realistic cosmological and astrophysical spacetimes are either asymptotically flat or have bounded curvature at spatial infinity, it is indeed a reasonable assumption to demand the gravity theory described by (3) to be ghost-free around Minkowski (Of course, we exclude the discussion of late time effects in some models of present dark energy leading to an unlimited growth of curvature in future. This is another issue indeed, and there are constructions of nonlocal gravity theories with both analytic and nonanalytic form factors that affect the IR nature of gravity beyond that of the Einstein gravity with an exact cosmological constant [57-61].). To obtain ghost-free form factors, the method is to obtain the second-order perturbation of the action (3) around the Minkowski background. To be explicit, we consider the general metric perturbation around a chosen background (in this section our discussion is limited

目前该引力理论由无穷集合 (f_{mR}, f_{mC}) 给出，包含无穷多个任意参数。但当我们要求理论无鬼时，形状因子的选择自由度会大幅降低 (4)。接下来需要解决的问题是: 理论需要在哪个背景下满足无鬼条件，以及如何得到形状因子的结构。我们可以先在我们所知最简单的背景——闵氏时空下回答这些问题。我们要求理论在闵氏时空附近无鬼，不仅因为闵氏是最简单的时空，还因为我们认为任何弯曲时空都必须局域等价于闵氏时空。这意味着在极短长度尺度或极高能量尺度 (但仍远低于局域洛伦兹不变性可能破缺的能标) 下，时空近似为闵氏时空。另一方面，大多数现实的宇宙学和天体物理时空要么是渐近平直的，要么在空间无穷远处曲率有界，因此要求由 (3) 描述的引力理论在闵氏时空附近无鬼是合理的假设 (当然，我们在此不讨论某些当前暗能量模型中未来曲率无限增长的晚期效应，这确实是另一个问题，目前已有同时包含解析与非解析形状因子的非局域引力理论构造，这类理论会影响引力的红外性质，不同于带精确宇宙学常数的爱因斯坦引力 [57-61])。为得到无鬼的形状因子，方法是获取作用量 (3) 在闵氏背景下的二阶微扰。具体来说，我们考虑选定背景下的一般度扰动量 (本节我们的讨论局限于

to maximally symmetric spacetimes (MSSs) such as Minkowski, dS, and anti-dS) as $g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{M_p}h_{\mu\nu}$ and consider the four-dimensional York decomposition of the fluctuations in terms of scalar, vector, and tensor parts as

闵氏、德西特、反德西特这类极大对称时空 (MSS))，记为 $g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{M_p}h_{\mu\nu}$，并将涨落按标量、矢量、张量部分做四维约克分解，形式如下

$$h_{\mu\nu} = h^{\perp}_{\mu\nu} + \overline{\nabla}_\mu A_\nu + \overline{\nabla}_\nu A_\mu + \left(\overline{\nabla}_\mu \overline{\nabla}_\nu - \bar{g}_{\mu\nu}\overline{\Box}\right) B + \frac{1}{4}\bar{g}_{\mu\nu} h, \tag{5}$$

where $h^{\perp}_{\mu\nu}$ is the transverse and traceless tensor, A_μ being the transverse vector, B, h are the scalars [34, 35] . Around Minkowski the vector part does not contribute to the second-order action, and the scalars B, h combine into an effective scalar $\varphi = \overline{\Box} B - h$. As a result, we obtain the second-order perturbed action as

其中 $h^{\perp}_{\mu\nu}$ 是无迹横向张量，A_μ 是横向矢量，B, h 是标量 [34, 35] 。在闵氏背景下，矢量部分对二阶作用量没有贡献，标量 B, h 会合并为一个有效标量 $\varphi = \overline{\Box} B - h$ 。由此我们得到二阶微扰作用量为

$$\delta^{(2)} S_q^{\text{Nonlocal}} = \frac{1}{2} \int d^4x \sqrt{-\bar{g}} \left[-\frac{3M_p^2}{32} \varphi \Box \mathcal{O}_0 \varphi + \frac{M_p^2}{4} h^{\perp}_{\mu\nu} \Box \mathcal{O}_2 h^{\perp\mu\nu} \right], \tag{6}$$

where the differential operators $\mathcal{O}_1$ and $\mathcal{O}_2$ take the form [34, 62]

其中微分算符 $\mathcal{O}_1$ 和 $\mathcal{O}_2$ 的形式为 [34, 62]

(7)

$$\mathcal{O}_0 = \left(1 - \frac{6}{M_p^2} \Box \mathcal{F}_R(\Box)\right)$$

$$\mathcal{O}_2 = \left(1 + \frac{2}{M_p^2} \Box \mathcal{F}_C(\Box_s)\right)$$

Following the Weierstrass factorization theorem, if the kinetic differential operators above take the exponential form, we do not introduce any additional degrees of freedom and as such we completely avoid ghost modes:

根据魏尔斯特拉斯分解定理, 若上述动力学微分算符取指数形式, 我们就不会引入任何额外自由度, 因此可以完全避免鬼模:

$$\mathcal{O}_0 = \left(1 - \frac{\Box}{M^2}\right)^X e^{H_0(\Box_s)}, \ \mathcal{O}_2 = e^{H_2(\Box_s)}, \tag{8}$$

where $H_0(\Box_s)$ and $H_2(\Box_s)$ are the entire functions. And $X = 1$ corresponds to the formulation of theory with an additional propagating scalar (called "scalaron"), while $X = 0$ is the case without any additional propagating scalar.

其中 $H_0(\Box_s)$ 和 $H_2(\Box_s)$ 是整函数。$X = 1$ 对应存在额外传播标量 (称为“标量子”) 的理论形式, $X = 0$ 对应不存在任何额外传播标量的情况。

Following (8), the form factors take the following structure:

由 (8) 可得，形状因子满足如下结构:

$$\mathcal{F}_R(\Box_s) = \frac{M_p^2}{6} \frac{1 - \left(1 - \frac{\Box}{M^2}\right)^p e^{H_0(\Box_s)}}{\Box}, \tag{9}$$

$$\mathcal{F}_C(\Box_s) = \frac{M_p^2}{2} \frac{e^{H_2(\Box_s)} - 1}{\Box}.$$

With the above form factors, the graviton propagator as a function of the four-momentum square p^2 looks like [13, 30]

采用上述形状因子后，作为四动量平方 p^2 函数的引力子传播子形式为 [13, 30]

$$\Pi\left(p^2\right) \sim -\frac{P^{(2)}}{p^2 e^{H_2(-p^2)}} + \frac{P^{(0)}}{2p^2\left(1+\frac{p^2}{M^2}\right)e^{H_0(-p^2)}}, \tag{10}$$

where $P^{(2)}, P^{(0)}$ are spin projection operators [62]. If we require the Newtonian potentials Φ, Ψ of the theory (3) to be equal around Minkowski, we arrive at the condition [63]

其中 $P^{(2)}, P^{(0)}$ 是自旋投影算符 [62]。若我们要求理论 (3) 的牛顿势 Φ, Ψ 在闵氏背景下相等，就能得到条件 [63]

$$H_2\left(\Box_s\right) = H_0\left(\Box_s\right), \tag{11}$$

which is also known to give a non-singular solution for Φ [63]. It is often speculated that because of the non-singular Newtonian potential, the theory might potentially avoid black hole singularities [17].

已知该条件同样能给出 Φ 的非奇异解 [63]。人们通常推测，由于牛顿势非奇异，该理论有可能避免黑洞奇点 [17]。

Let us now understand if NLQG (3) including the cosmological constant term

现在我们来研究包含宇宙学常数项的 NLQG (3)

$$S_{q\Lambda}^{\text{Nonlocal}} = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_p^2 R + R\mathcal{F}_R\left(\Box_s\right)R + W_{\mu\nu\rho\sigma}\mathcal{F}_C\left(\Box_s\right)W^{\mu\nu\rho\sigma} - \Lambda\right]$$

(12)

is ghost-free. Again, we expect that it entirely depends on the structure of the form factors $\mathcal{F}_R\left(\Box_s\right)$ and $\mathcal{F}_C\left(\Box_s\right)$, which can be determined by computing second-order action for (12) around the MSS background

是否无鬼。我们再次认为，这完全取决于形状因子 $\mathcal{F}_R\left(\Box_s\right)$ 和 $\mathcal{F}_C\left(\Box_s\right)$ 的结构，二者可以通过计算 MSS 背景下 (12) 的二阶作用量得到

$$\bar{R} = 4\Lambda,\ \bar{R}_{\mu\nu} = \frac{\bar{R}}{4}\bar{g}_{\mu\nu},\ \bar{R}^{\mu}_{\nu\rho\sigma} = \frac{\bar{R}}{12}\left(\delta^{\mu}_{\rho}\bar{g}_{\nu\sigma} - \delta^{\mu}_{\sigma}\bar{g}_{\nu\rho}\right). \tag{13}$$

With the York decomposition of the metric fluctuation (5), we obtain the second-order action of (12) as

借助度量涨落 (5) 的约克分解，我们得到 (12) 的二阶作用量为

$$\begin{aligned}\delta^{(2)}S_{q\Lambda}^{\text{Nonlocal}} = &\frac{1}{2}\int d^4x\sqrt{-\bar{g}}\left[-\frac{3M_p^2}{32}\varphi\mathcal{O}_{0\Lambda}\left(\bar{\Box}+\frac{\bar{R}}{3}\right)\varphi\right.\\ &\left.+\frac{M_p^2}{4}h^{\perp}_{\mu\nu}\mathcal{O}_{2\Lambda}\left(\bar{\Box}-\frac{4\Lambda}{6}\right)h^{\perp\mu\nu}\right],\end{aligned} \tag{14}$$

where

其中

$$\mathcal{O}_\Lambda = \left(1 - \frac{\overline{\Box}}{M^2}\right)^p e^{H_{0\Lambda}(\overline{\Box}_s,\Lambda_s)} = 1 + f_{0R}\frac{8\Lambda}{M_p^4} - \frac{2}{M_p^2}\left(3\overline{\Box} + 4\Lambda\right)\mathcal{F}_R\left(\overline{\Box}_s\right),$$

$$\mathcal{O}_{2\Lambda} = e^{H_{2\Lambda}(\overline{\Box}_s,\Lambda_s)} = 1 + \frac{8\Lambda}{M_p^4} f_{0R} + \frac{2}{M_p^2}\left(\overline{\Box} - \frac{\Lambda}{3}\right)\mathcal{F}_C\left(\overline{\Box}_s + \frac{4\Lambda_s}{3}\right),$$

(15)

where $\Lambda_s = \frac{\Lambda}{\mathcal{M}_s^2}$. Here $p = 0, 1$ corresponds to having additional no scalaron and a scalaron degree of freedom, respectively. From (15), we can work out the structure of form factors. To have them analytic, one must appropriately fix the entire functions $H_{0\Lambda}, H_{2\Lambda}$ such that in the limit $\Lambda \to 0$ we should get back the form factors of Minkowski spacetime (9). The main lesson we can learn from (15) is that if we demand the theory to be ghost-free around dS or anti-dS, we are forced to consider form factors that not only depend on the d'Alembertian operator but should also be a function of Λ in the appropriate form. On the other hand, what we learn here is that the form factors are background dependent, and eventually the nonlocal gravity theory we started with (3) is background dependent as well. If we want the theory to be background independent, we are somehow forced to consider higher curvature terms. Indeed around curved spacetime, the cubic, quartic, and even higher order curvature terms become relevant and unavoidable. This is not something surprising and here is an intuitive explanation. As we can see, (3) is AID extension of Stelle gravity (1). But logically if we add $W_{\mu\upsilon\rho\sigma}\Box_s W^{\mu\upsilon\rho\sigma}$, we should equally add $RW_{\mu\upsilon\rho\sigma}W^{\mu\upsilon\rho\sigma}$ because both are sixth-order derivative terms. The same logic applies to any N th-order term, and we cannot omit one term in favor of other. This means the construction of the action (3) is incomplete, and surely we must add more terms to it for consistency and to construct a theory that is ghost-free around curved spacetime. In the several studies in the past, NLQG (3) is claimed to be the theory that is super-renormalizable and a potential candidate to be UV complete [16, 27, 64, 65] , but all that is around Minkowski. For any practical application, we need a ghost-free theory in curved spacetime, and as we saw in the simplest example of dS and AdS spacetimes, we must add higher curvature nonlocal terms. To ascertain the same fact, we are going to further explain this in the context of inflationary scenario from (3). Since the action (3) has infinite derivatives, one thinks that it is impossible to solve the equations of motion (see [30,33]). However, due to the structure of the action (3), we can solve the equations of motion by proposing an ansatz that leads to recursive relations with the action of d'Alembertian operation on curvature quantities. In the context of cosmological Friedmann-Lemaître-Robertson-Walker (FLRW) backgrounds, we do not have to worry about any contributions to the equations of motion coming from the variation of nonlocal Weyl square term in (3). This simplifies significantly our quest for solving equations of motion, and by using the simple eigenvalue equation

其中 $\Lambda_s = \frac{\Lambda}{\mathcal{M}_s^2}$ 。此处 $p = 0, 1$ 分别对应无额外标量子和存在一个额外标量子自由度的情况。根据式 (15)，我们可以推导出形状因子的结构。若要求形状因子解析，必须适当固定整函数 $H_{0\Lambda}, H_{2\Lambda}$ ，使得在极限 $\Lambda \to 0$ 下，我们能重新得到闵氏时空的形状因子 (9)。从式 (15) 我们能得到的核心结论是: 如果要求理论在德西特或反德西特背景下无鬼，那么形状因子不仅必须依赖于达朗贝尔算符，还需要以恰当的形式成为 Λ 的函数。另一方面，我们在此发现形状因子是依赖于背景的，因此我们最初采用的非局域引力理论 (3) 也同样依赖于背景。如果要求理论背景无关，我们就不得不引入更高曲率项。的确，在弯曲时空周围，三次、四次乃至更高阶的曲率项都会变得相关且不可避免。这并不出人意料，我们可以给出一个直观的解释。可见式 (3) 是施泰勒引力 (1) 的解析无限导数 (AID) 推广。但逻辑上，如果我们已经引入了 $W_{\mu\upsilon\rho\sigma}\Box_s W^{\mu\upsilon\rho\sigma}$ ，就同样应当引入 $RW_{\mu\upsilon\rho\sigma}W^{\mu\upsilon\rho\sigma}$ ，因为二者都是六阶导数项。相同的逻辑适用于任意 N 阶项，我们不能偏废任意一项。这说明作用量 (3) 的构造是不完备的，为了自洽性，为了构造一个在弯曲时空下无鬼的理论，我们确实必须补充更多项。在过去的多项研究中，NLQG(3) 被宣称是超可重整化的理论，也是紫外完备的潜在候选者 [16, 27, 64, 65] ，但所有这些结论都是在闵氏背景下得到的。对于任何实际应用，我们都需要一个在弯曲时空下无鬼的理论，正如我们在德西特和反德西特时空的简单例子中看到的，我们必须引入更高阶的曲率非局域项。为了佐证这一结论，我们接下来会在式 (3) 的框架下，结合暴胀场景进一步解释这一点。由于作用量 (3) 包含无限多导数，人们通常认为无法求解其运动方程 (见文献 [30,33])。然而，得益于作用量 (3) 自身的结构，我们可以通过假设一个拟设来求解运动方程，该拟设会给出达朗贝尔算符作用在曲率量上的递推关系。在宇宙学弗里德曼-勒梅特-罗伯逊-沃尔克 (FLRW) 背景下，我们无需担心式 (3) 中非局域外尔平方项变分对运动方程的任何贡献。这极大简化了我们求解运动方程的工作，利用简单的本征方程

$$\Box R = M^2 R, \tag{16}$$

we can solve completely equations of motion of (3) for FLRW backgrounds with the following simple conditions on the form factor $\mathcal{F}_R(\Box_s)$ evaluated at $\Box = M^2$:

我们可以在如下关于形状因子 $\mathcal{F}_R(\Box_s)$ 的简单条件下，完全求解 FLRW 背景下式 (3) 的运动方程，其中形状因子在 $\Box = M^2$ 处求值:

$$\mathcal{F}_R\left(\frac{M^2}{\mathcal{M}_s^2}\right) = f_0 = \frac{M_p^2}{6M^2},\ \mathcal{F}_R^\dagger\left(\frac{M^2}{\mathcal{M}_s^2}\right) = 0, \tag{17}$$

where † denotes the derivative with respect to the argument. If we consider the form factor (9), then (17) exactly implies $H_0(0) = 0$ and $p = 1$. Clearly, this means we need the NLQG (3) with a propagating scalaron degree of freedom to have (16) as a background FLRW solution. As a matter of fact, (16) is exactly the trace equation of the local $R + R^2$ gravity (2) which solution just provides the Starobinsky inflationary model [2, 30] . If we require inflationary scenario to be stable against perturbations, we have to require the form factors to be ghost-free. Again to determine this, we have to compute the second-order action for (3), but in this case it is convenient to define the metric fluctuations in terms of $1 + 3$ decomposition as

其中 † 表示对宗量求导。若我们考虑形状因子 (9)，那么式 (17) 恰好推出 $H_0(0) = 0$ 和 $p = 1$ 。显然，这说明我们需要带有传播标量自由度的 NLQG(3)，才能让式 (16) 成为 FLRW 背景解。事实上，式 (16) 正是定域 $R + R^2$ 引力 (2) 的迹方程，该方程的解恰好就是 Starobinsky 暴胀模型 [2, 30] 。如果我们要求暴胀景像对微扰稳定，就必须要求形状因子无鬼。为验证这一点，我们同样需要计算式 (3) 的二阶作用量，这种情况下按 $1 + 3$ 分解来定义 metric 涨落会更方便，分解形式如下:

$$ds^2 = a^2(\tau)\left(-(1+2\Phi)\,d\tau^2 + \left((1-2\Psi)\,\delta_{ij} + h_{ij}\right)dx^i dx^j\right), \tag{18}$$

where Φ and Ψ are the Bardeen potentials and h_{ij} is a transverse and traceless tensor. The two Bardeen potentials are constrained during inflation as

其中 Φ 和 Ψ 是 Bardeen 势，h_{ij} 是横迹零张量。两个 Bardeen 势在暴胀过程中满足如下约束:

$$\left[f_0\bar{R}_{\rm dS} + \left(\overline{\Box} - \frac{\bar{R}_{\rm dS}}{6}\right)\mathcal{F}_C\left(\overline{\Box}_s + \frac{\bar{R}_{\rm dS}}{2\mathcal{M}_s^2}\right)\right](\Phi + \Psi) = 0. \tag{19}$$

We derive (19) after linearizing equations of motion of (3) around the inflationary background satisfying (16) and then applying the quasi-dS (slow-roll) approximation, i.e., $\bar{R}_{\rm dS} \gg 3M^2$ and slowly changing during inflation (here $\bar{R}_{\rm dS}$ is the value of Ricci scalar in this epoch) [30]. Curiously in (19), we can notice that the form factor $\mathcal{F}_C(\Box_s)$ is involved. In the case of local R^2 inflation, we know that $\Phi + \Psi \approx 0$ during inflation [66, 67] , but to have the same thing to hold in the context of NLQG (3), we must need the operator acting on $\Phi + \Psi$ in (19) to be exponent of an entire function, and at the same time we need to maintain the analyticity property of the form factor $\mathcal{F}_C(\Box)$. This implies

我们将式 (3) 的运动方程在满足式 (16) 的暴胀背景线性化，再应用拟 dS(慢滚) 近似，也就是 $\bar{R}_{\rm dS} \gg 3M^2$ 且在暴胀过程中缓慢变化 (此处 $\bar{R}_{\rm dS}$ 是该阶段里奇标量的值)[30]，最终推导出式 (19)。有意思的是，我们可以注意到式 (19) 中包含形状因子 $\mathcal{F}_C(\Box_s)$ 。在定域 R^2 暴胀的情况中，我们知道暴胀过程中 $\Phi + \Psi \approx 0$ 满足 [66, 67] ，但要让这个结论在 NLQG(3) 的框架下同样成立，我们必须要求式 (19) 中作用在 $\Phi + \Psi$ 上的算符是整函数的指数，同时还需要保持形状因子 $\mathcal{F}_C(\Box)$ 的解析性。由此可以推出:

$$\mathcal{F}_C(\Box_s) = f_0\bar{R}_{\rm dS}\frac{e^{\gamma_T\left(\Box_s - \frac{\bar{R}_{\rm dS}}{3\mathcal{M}_s^2}\right)} - 1}{\left(\overline{\Box}_{\rm dS} - \frac{2\bar{R}_{\rm dS}}{3\mathcal{M}_s^2}\right)}. \tag{20}$$

We can see that the form factor (20) contains the dependence on the background again similar to the case of exact dS (15), but here we have the dependence through $\bar{R}_{\rm dS}$ which is treated to be nearly constant, but in practice Ricci scalar is not exactly constant but rather slowly varying according to (16). This means the action (3) cannot be the fundamental action that describes inflation, but rather it must be an effective version of some more fundamental theory of nonlocal gravity. Furthermore, if we compute the second-order action of (3) for tensor perturbation with the form factor (20), we obtain

我们可以看到，和精确 dS 的情况 (15) 类似，形状因子 (20) 同样依赖背景，只不过这里的依赖是通过 $\bar{R}_{\rm dS}$ 实现的——$\bar{R}_{\rm dS}$ 被近似为近常数，但实际上根据式 (16)，里奇标量并非严格常数，而是缓慢变化的。这说明作用量 (3) 不可能是描述暴胀的基本作用量，它只能是某个更基本的非局域引力理论的有效版本。此外，如果我们对带形状因子 (20) 的式 (3) 计算张量涨落的二阶作用量，可得:

$$\delta^{(2)}_{(t)} S_q^{\rm Nonlocal} = \int d^4x\sqrt{-g}\left[h_{ij}e^{\gamma_T\left(\frac{\bar{\Omega}_{\rm dS}}{\mathcal{M}_s^2}-\frac{\bar{R}_{\rm dS}}{3\mathcal{M}_s^2}\right)}\left(\overline{\Omega}_{\rm dS}-\frac{\bar{R}_{\rm dS}}{6}\right)h^{ij}\right]. \tag{21}$$

The above action (21) exactly tells us that NLQG (20) is indeed the ghost-free form factor in the context of inflationary background. Substituting (20) in the action (3), we can notice terms like

上述作用量 (21) 清楚地表明，在暴胀背景的框架下，NLQG(20) 确实是无鬼形状因子。将式 (20) 代入作用量 (3)，我们可以得到如下形式的项:

$$\frac{\bar{R}_{\rm dS}}{\mathcal{M}_s^2}W_{\mu\upsilon\rho\sigma}W^{\mu\upsilon\rho\sigma},\ \frac{\bar{R}_{\rm dS}}{\mathcal{M}_s^2}W_{\mu\upsilon\rho\sigma}\Box_s W^{\mu\upsilon\rho\sigma},\ \left(\frac{\bar{R}_{\rm dS}}{\mathcal{M}_s^2}\right)^2 W_{\mu\upsilon\rho\sigma}\Box_s W^{\mu\upsilon\rho\sigma},$$
$$\frac{\bar{R}_{\rm dS}}{\mathcal{M}_S^2}W_{\mu\upsilon\rho\sigma}\Box_s^2 W^{\mu\upsilon\rho\sigma}\ldots \tag{22}$$

which again clearly indicate the necessity of introducing higher order curvature terms; otherwise we end up with an action that has specific background curvature dependence. Another indication why (20) in (3) is not a consistent picture is because such a theory would possibly (if $\bar{R}_{dS}\gtrsim\mathcal{M}_s^2$) lead to larger production of scalarons decaying into gravitons after the end of inflation according to recent study [68] (note that in the local $R+R^2$ gravity, the scalaron decay into two gravitons is suppressed [66]). This is because if $\bar{R}_{dS}$ is treated as a fixed constant, its effect will be there even after inflation ends. In the next section we present a consistent extension of the NLQG (3) with the higher curvature nonlocal terms where we can see that the form factor (20) emerges naturally at the linearized level in the quasi-dS approximation.

这再次清晰表明引入高阶曲率项的必要性；否则我们最终得到的作用量会带有特定的背景曲率依赖。(20) 放在 (3) 中不自洽的另一项证据是: 根据近期研究 [68]，这类理论有可能 (若 $\bar{R}_{dS}\gtrsim\mathcal{M}_s^2$) 导致暴胀结束后产生更多衰变到引力子的标量子 (注意在定域 $R+R^2$ 引力中，标量子衰变到双引力子的过程是被压低的 [66])。原因在于，如果 $\bar{R}_{dS}$ 被当作固定常数，它的效应即使在暴胀结束后依然存在。在下一节中，我们会给出 NLQG(3) 的一个自洽推广，其中引入了高阶曲率非局域项，可以看到形状因子 (20) 会在拟 dS 近似的线性水平自然出现。

Let us now discuss scalar perturbations. Considering $\Phi+\Psi\approx 0$ during inflation, the second-order action of (31) for the scalar perturbations becomes [31]

现在我们来讨论标量微扰。考虑暴胀期间的 $\Phi+\Psi\approx 0$ ，式 (31) 中标量微扰的二阶作用量为 [31]

$$\delta^{(2)}_{(s)}S = \frac{1}{2f_0\bar{R}_{\rm dS}}\int d^4x\sqrt{-\bar{g}}\Upsilon\frac{\mathcal{W}\left(\overline{\Box}_s\right)}{\mathcal{F}_R\left(\overline{\Box}_s\right)}\left(\overline{\Box}_{\rm dS}-M^2\right)\Upsilon, \tag{23}$$

where $\Upsilon = 2f_0\bar{R}_{\rm dS}\Psi$ is the canonical variable, and it is related to the curvature perturbation as $\Upsilon\approx -2\epsilon f_0\bar{R}_{\rm dS}\mathcal{R}$. The operator $\mathcal{W}\left(\overline{\Box}_s\right)$ is given by

其中 $\Upsilon = 2f_0\bar{R}_{\mathrm{dS}}\Psi$ 是正则变量，它和曲率微扰满足关系 $\Upsilon \approx -2\varepsilon f_0\bar{R}_{\mathrm{dS}}\mathcal{R}$ 。算符 $\mathcal{W}(\Box_s)$ 由下式给出

$$\mathcal{W}(\Box_s) = 3\mathcal{F}_R(\Box_s) + \left(\bar{R}_{\mathrm{dS}} + 3M^2\right)\frac{\mathcal{F}_R(\Box_s) - f_0}{\Box - M^2}. \tag{24}$$

From (23) we can verify that the kinetic term of Υ has one real zero corresponding to $\overline{\Box}_{\mathrm{dS}} = M^2$. This means there is one propagating degree of freedom (i.e., scalaron). If any other degrees of freedom exist, they must arise from zeros of the operator $\mathcal{W}(\Box_s)$ (24) . Considering $\mathcal{W}(\Box_s) = 3f_0e^{\gamma_0\left(\Box_s+\frac{\bar{R}_{\mathrm{dS}}}{3\mathcal{M}_s^2}\right)}$, where γ_0 is an entire function of $\Box_s + \frac{\bar{R}_{\mathrm{dS}}}{3\mathcal{M}_s^2}$, we get no zeros in the entire complex plane. But the consequence is that the form factor $\mathcal{F}_R(\Box_s)$ dependence on the background quantity $\bar{R}_{\mathrm{dS}}$ is unavoidable [19, 29] :

由式 (23) 我们可以验证，Υ 的动能项存在一个对应于 $\overline{\Box}_{\mathrm{dS}} = M^2$ 的实零点，这说明存在一个传播自由度 (即标量子)。若存在其他自由度，它们必然来自算符 $\mathcal{W}(\Box_s)$ (24) 的零点。考虑 $\mathcal{W}(\Box_s) = 3f_0e^{\gamma_0\left(\Box_s+\frac{\bar{R}_{\mathrm{dS}}}{3\mathcal{M}_s^2}\right)}$ ，其中 γ_0 是 $\Box_s + \frac{\bar{R}_{\mathrm{dS}}}{3\mathcal{M}_s^2}$ 的整函数，我们可知其在整个复平面上不存在零点。但由此得到的结论是，形状因子 $\mathcal{F}_R(\Box_s)$ 对背景量 $\bar{R}_{\mathrm{dS}}$ 的依赖是不可避免的 [19, 29] :

$$\mathcal{F}_R(\Box_s) \equiv \mathcal{F}_R\left(\Box_s, \bar{R}_{\mathrm{dS}}\right) = f_0\frac{e^{\gamma_0\left(\Box_s+\frac{\bar{R}_{\mathrm{dS}}}{3\mathcal{M}_s^2}\right)}\left(\Box_s - \frac{M^2}{\mathcal{M}_s^2}\right) + \left(\bar{R}_{\mathrm{dS}} + 3M^2\right)}{3\Box + \bar{R}_{\mathrm{dS}}}. \tag{25}$$

This choice of form factor (25) again leads to background dependence in the action (31) similar to (15) and (20) through the factor R_{dS} . But if we take (9) instead, we completely avoid any background curvature dependence, and also we do not have to worry about the Minkowski limit. Calculating $\mathcal{W}(\Box_s)$ for (9), we obtain

形状因子的选择 (25) 再次通过因子 R_{dS} 使得作用量 (31) 出现类似式 (15) 和 (20) 的背景依赖。但如果我们改用式 (9)，则可以完全避免任何背景曲率依赖，也无需担心闵氏极限。对式 (9) 计算 $\mathcal{W}(\Box_s)$ ，我们得到

$$\mathcal{W}(\Box_s) = -f_0\bar{R}_{\mathrm{dS}}\left(\frac{1 - e^{\gamma_S(\Box_s)}}{\Box}\right) + 3f_0e^{\gamma_S(\Box_s)}. \tag{26}$$

The zeros of $\mathcal{W}(\Box_s)$ can be obtained by solving

$\mathcal{W}(\Box_s)$ 的零点可以通过求解下式得到

$$\mathcal{W}\left(\frac{Z}{\mathcal{M}_s^2}\right) = 0 \Rightarrow \bar{R}_{\mathrm{dS}}\left(\frac{1 - e^{\gamma_S\left(\frac{Z}{\mathcal{M}_s^2}\right)}}{Z}\right) = 3e^{\gamma_S\left(\frac{Z}{\mathcal{M}_s^2}\right)}. \tag{27}$$

To solve (27), let us consider the following entire function for simplicity:

为求解式 (27)，为简化起见我们考虑如下整函数:

$$\gamma_S(\Box_s) = \alpha_1\Box_s\left(\Box_s - \frac{M^2}{\mathcal{M}_s^2}\right). \tag{28}$$

Substituting (28) into (27), we get no real solutions but instead complex conjugate ones:

将式 (28) 代入式 (27)，我们没有得到实解，仅得到复共轭解:

$$
\begin{aligned}
Z = & \frac{M^2}{2} + \frac{\mathcal{M}_s^2}{2}\left[(2\pi + 4q\pi)^2 + \left(\frac{M}{\mathcal{M}_s}\right)^8\right]^{1/4} \\
& \times\left\{\cos\left[\frac{1}{2}\operatorname{Arg}\left[4\pi i\left(q+\frac{1}{2}\right)+\frac{M^4}{\mathcal{M}_s^4}\right]\right]\right. \\
& \left.+ i\sin\left[\frac{1}{2}\operatorname{Arg}\left[4\pi i\left(q+\frac{1}{2}\right)+\frac{M^4}{\mathcal{M}_s^4}\right]\right]\right\} \\
& \approx |_{M^2\ll\mathcal{M}_s^2} \pm \mathcal{M}_s^2\sqrt{q+\frac{1}{2}}\,(1\pm i)\,,
\end{aligned} \tag{29}
$$

where $q \geq 1$ is a positive integer. It was known from works based on string field theory models [69-72] that complex conjugate poles give (classical) degrees of freedom for a coupled system of scalar fields having both positive and negative kinetic terms in equal numbers. The question is whether these degrees of freedom are physical and contribute to inflationary correlators. Nonlocal scalar field theories with infinitely many complex conjugate poles are studied in [73, 74] with the following type of Lagrangians:

其中 $q \geq 1$ 是正整数。从基于弦场论模型的研究 [69-72] 可知，复共轭极点为正负动能项数目相等的耦合标量场系统提供了 (经典) 自由度。问题在于这些自由度是否是物理的，是否会对暴胀关联函数产生贡献。具有无穷多复共轭极点的非局部标量场论在 [73, 74] 中已有研究，这类研究采用如下拉格朗日量形式:

$$
\mathcal{L}_\phi = \frac{1}{2}\phi\left(e^{\gamma_\phi(\Box_s)} - 1\right)\phi - V(\phi)\,, \tag{30}
$$

where γ_ϕ being an arbitrary entire function. It was found that the optical theorem both at the tree level [73, 74] and at the one-loop level [75] is satisfied despite the presence of complex conjugate poles. This is due to the exact cancellation of contributions from degrees of freedom corresponding to complex conjugate poles. Similar to nonlocal theories like (30), complex conjugate poles also appear in the context of Lee-Wick theories where one can project away the states using new quantum field theory prescriptions. As several investigations suggest [9, 76-84], we may disregard these states as unphysical (Alternatively, the imaginary part of the pole can be made small enough (by a contrived choice of entire function) to avoid any classical instabilities [85]. But such a choice of entire function necessarily should depend on the background value of $\bar{R}_{\rm dS}$.).

其中 γ_ϕ 为任意整函数。研究发现，尽管存在复共轭极点，树图阶 [73, 74] 和单圈阶 [75] 的光学定理都成立，这是因为复共轭极点对应自由度的贡献恰好完全抵消。和式 (30) 这类非局部理论类似，复共轭极点也会出现在 Lee-Wick 理论中，在该理论中人们可以通过新的量子场论规则投影去掉这些态。多个研究表明 [9, 76-84]，我们可以将这些态视为非物理态而忽略 (或者，通过人为选择整函数可以让极点的虚部足够小，避免任何经典不稳定性 [85]。但这种整函数的选择必然依赖于 $\bar{R}_{\rm dS}$ 的背景值。)

Several of these studies are about nonlocal scalar field theories in Minkowski spacetime. In the context of gravitational action (3) with (9), we have no degrees of freedom corresponding to complex conjugate poles around Minkowski spacetime. We can also notice that in the limit $\bar{R}_{\rm dS} \to 0$, we have $\mathcal{W}(\Box_s) \to 3f_0 e^{\gamma_S(\Box_s)}$

(see (26)) that confirm there exists only scalaron in addition to massless graviton. It is justifiable to use the form factor (9) in the context of inflation because when we quantize inflationary fluctuations, we impose adiabatic vacuum initial conditions for them deep inside the Hubble radius $k \gg aH$, where $H = \frac{\dot{a}}{a}$ is the Hubble parameter of a spatially flat FLRW model [19, 67] . Therefore, in the limit $k \gg aH$, we can ignore all the complex conjugate modes (29) by setting initial conditions for them to be zero that implies inflationary correlations are only sourced by scalaron and massless graviton modes.

其中多项研究针对的是闵氏时空下的非局部标量场理论。对于结合 (9) 式的引力作用量 (3)，我们在闵氏时空附近不存在对应复共轭极点的自由度。还可以注意到，在极限 $\bar{R}_{\mathrm{dS}} \to 0$ 下，我们得到 $\mathcal{W}(\Box_s) \to 3f_0 e^{\gamma_S(\Box_s)}$ (见 (26) 式)，这证实除无质量引力子外仅存在标量子。在暴胀研究中使用形状因子 (9) 是合理的，因为当我们对暴胀涨落做量子化时，会在哈勃半径深处为其施加绝热真空初始条件 $k \gg aH$ ，其中 $H = \frac{\dot{a}}{a}$ 是空间平坦 FLRW 模型的哈勃参数 [19, 67] 。因此，在极限 $k \gg aH$ 下，我们可以通过将所有复共轭模式的初始条件设为零来忽略它们 (即 (29) 式中的模式)，这意味着暴胀关联仅由标量子和无质量引力子模式产生。

In summary, we learned that the NLQG action (3) is incomplete if we consider curved spacetime. This straightforwardly leads us to formulate a consistent quantum gravity theory that goes beyond (3). Second, applying the framework of R^2 -like inflation, we have learned that we must fully avoid any background dependence of the form factors that we can only do by considering higher curvature nonlocal terms. The third point is if we have constructed a ghost-free nonlocal gravity theory for some choice of form factors in the Minkowski spacetime, we can expect the appearance of complex conjugate pairs as poles in the propagators for perturbed modes. Due to the fact that these modes do not exist in the local Minkowski limit, we might set their initial conditions to zero, but understanding them requires further development of quantum field theory in curved spacetime (As a side remark we would like to point out that quantum field theory of nonlocal theories around Minkowski is another subject of investigation because of possible causality violation around scales shorter than nonlocality scale and issues with the Wick rotation. However, it was noted that microcausality violation may not be a problem because we can never probe that regime [86, 87] , and also inflation happens to be at length scales larger than the nonlocality scale [30]. The problem of Wick rotation in nonlocal field theory has also been successfully addressed with a new prescription of contour integration (see [85, 88, 89] , and the references therein).).

综上所述，我们发现如果考虑弯曲时空，NLQG 作用量 (3) 是不完备的，这自然推动我们构造一个超出 (3) 式的自洽量子引力理论。其次，应用 R^2 类暴胀框架后我们认识到，必须完全避免形状因子依赖背景，这一点只有通过考虑高阶曲率非局部项才能实现。第三点: 如果我们在闵氏时空下针对某组形状因子构造了一个无鬼非局部引力理论，可以预期微扰模式传播子中会出现复共轭对极点。由于这些模式在局域闵氏极限中不存在，我们可以将它们的初始条件设为零，但要理解它们还需要进一步发展弯曲时空量子场论 (补充说明: 我们需要指出，闵氏时空背景下非局部理论的量子场论本身就是一个研究课题，这是因为非局域尺度以下的尺度可能存在因果性破坏，还存在维克转动相关问题。但已有研究指出，微观因果性破坏可能并不构成问题，因为我们永远无法探测该区域 [86, 87] ，此外暴胀发生在远大于非局域尺度的长度尺度上 [30]。非局部场论中的维克转动问题也已经通过一种新的围道积分处方得到了成功解决 (见 [85, 88, 89] 及其中参考文献)。)。

Generalized Nonlocal Gravity and R^2 -Like Inflation

广义非局域引力与 R^2 类暴胀

As we discussed in the previous sections, one needs to go beyond the (3) to have a consistent ghost-free theory for the dynamical backgrounds such as inflation. In this section, we discuss the generalized nonlocal gravity which is compatible with R^2 -like inflation as a background solution [32]. Note that local R^2 -inflation is an important guiding principle to build a consistent quantum theory of gravity. The action for generalized nonlocal gravity is

正如我们在前几节讨论的，对于暴胀这类动力学背景，需要超出式 (3) 才能得到一个自洽的无鬼场理论。本节我们讨论与 R^2 类暴胀背景解相容的广义非局域引力 [32]。值得注意，局域 R^2 暴胀是构建自洽量子引力理论的重要指导原则。广义非局域引力的作用量为

$$
\begin{aligned}
S_H^{\text{Nonlocal}} &= \frac{1}{2}\int d^4x\sqrt{-g}\Bigg(M_p^2 R + \Bigg[R\mathcal{F}_R(\Box_s)R + \left(\frac{M_p^2}{2\mathcal{M}_s^2} + f_0 R_s\right) \\
&\times W_{\mu\upsilon\rho\sigma}\mathcal{F}_W(\Box_s, R_s)W^{\mu\upsilon\rho\sigma} \\
&+ \frac{f_0\lambda_c}{\mathcal{M}_s^2}\mathcal{L}_1(\Box_s)R\mathcal{L}_2(\Box_s)R\mathcal{L}_3(\Box_s)R \\
&+ \frac{f_0\lambda_R}{\mathcal{M}_s^2}\mathcal{D}_1(\Box_s)R\mathcal{D}_2(\Box_s)W^{\mu\upsilon\gamma\lambda}\mathcal{D}_3(\Box_s)W_{\mu\upsilon\gamma\lambda} \\
&+ \frac{f_0\lambda_W}{\mathcal{M}_s^2}\mathcal{C}_1(\Box_s)W_{\mu\upsilon\rho\sigma}\mathcal{C}_2(\Box_s)W^{\mu\upsilon\gamma\lambda}\mathcal{C}_3(\Box_s)W_{\gamma\lambda}{}^{\rho\sigma}\Bigg] + \cdots\Bigg),
\end{aligned}
$$

(31)

which is a significant extension of (3) in the scope of R^2 -like inflation. Here $R_s = \frac{R}{\mathcal{M}_s^2}$ and $\cdots$ represent higher order curvature terms which are only relevant for 4-point cosmological correlations and beyond. In this chapter, we restrict to results of up to 3-point inflationary correlations. In (31) we write all possible nonlocal terms involving Ricci scalar and Weyl tensor. Let us look into these in more detail. The form factors in the first line of (31) can be fixed as

这是 R^2 类暴胀框架下对式 (3) 的重要推广。此处 $R_s = \frac{R}{\mathcal{M}_s^2}$ 和 $\cdots$ 代表高阶曲率项，仅对 4 阶及更高阶宇宙关联函数有意义。本章我们仅讨论至多 3 阶暴胀关联函数的结果。在式 (31) 中我们写出了所有包含里奇标量和外尔张量的可能非局域项，下面我们对此展开详细分析。式 (31) 第一行中的形状因子可以固定为

$$
\begin{aligned}
\mathcal{F}_R &= f_0 M^2 \frac{1 - \left(1 - \frac{\Box}{M^2}\right)e^{\gamma_S(\Box_s)}}{\Box}, \\
\mathcal{F}_W(\Box_s, R_s) &= \frac{e^{\gamma_T\left(\Box_s - \frac{2}{3}R_s\right)} - 1}{\Box_s - \frac{2}{3}R_s}
\end{aligned}
\tag{32}
$$

where $\gamma_S(\Box_s)$ and $\gamma_T(\Box_s)$ are the entire functions which can be finite degree polynomials or functions of polynomials, so that the theory has only a finite number of free parameters. With a careful observation, we can deduce that (32) very much looks like (20) except that this form factor is the function of Ricci scalar rather than of a particular value of it like in (20). As is easy to see from (31), our requirement of having R^2 -like inflation demands

其中 $\gamma_S(\Box_s)$ 和 $\gamma_T(\Box_s)$ 是整函数，可以是有限次多项式或多项式函数，因此该理论仅含有限个自由参数。仔细观察可以推导出，式 (32) 和式 (20) 形式非常相似，区别仅在于此处形状因子是里奇标量的函数，而非式 (20) 中里奇标量取特定定值的函数。从式 (31) 不难看出，我们对 R^2 类暴胀的要求给出

$$\gamma_S\left(\frac{M^2}{\mathcal{M}_s^2}\right)=\mathcal{L}_i\left(\frac{M^2}{\mathcal{M}_s^2}\right)=\mathcal{D}_1\left(\frac{M^2}{\mathcal{M}_s^2}\right)=0. \tag{33}$$

Obviously the fourth line of (31) is irrelevant for inflationary background since the background Weyl tensor is zero in FLRW. We omit Ricci tensor terms because inflation is by definition quasi-dS $\bar{R}_{\mu\upsilon}\approx\frac{\bar{R}}{4}\bar{g}_{\mu\upsilon}$; thus the terms involving it can be neglected. Of course, Ricci tensor-dependent terms might be important for curvature scales exceeding the inflationary ones toward the Planck scale.

显然，式 (31) 第四行对暴胀背景没有影响，因为 FLRW 度规下背景外尔张量为零。我们省略了里奇张量项，因为根据定义暴胀是拟 de Sitter $\bar{R}_{\mu\upsilon}\approx\frac{\bar{R}}{4}\bar{g}_{\mu\upsilon}$ ，因此包含里奇张量的项可以忽略。当然，当曲率尺度超过暴胀尺度接近普朗克尺度时，依赖里奇张量的项可能会很重要。

The cubic terms in (31) (i.e., second, third, and fourth lines) form all the possible combinations after judicious weeding of total derivatives using the following nonlocal generalization of curvature identities derived in [5, 7]

式 (31) 中的三次项 (即第二、第三和第四行) 是利用在 [5, 7] 中推导得到的曲率恒等式的非局域推广，对全导数进行合理剔除后得到的所有可能组合

$$R_\lambda^\sigma W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\lambda}=\frac{R}{4}W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}$$

$$\Rightarrow R_\lambda^\sigma\mathcal{O}(\Box_s)\,W_{\mu\nu\rho\sigma}\mathcal{O}(\Box_s)\,W^{\mu\nu\rho\lambda}=\frac{R}{4}\mathcal{O}(\Box_s)\,W_{\mu\nu\rho\sigma}\mathcal{O}(\Box_s)\,W^{\mu\nu\rho\sigma}.$$

(34)

We can notice in (31) that we do not have any terms with single covariant derivatives because we can eliminate all those terms by adding arbitrary number of total derivatives. For example, suppose we have a term of the form $\mathbb{R}\nabla_\mu\mathbb{R}\nabla^\mu\mathbb{R}$ in the action. Then we can eliminate this term by adding a total derivative of the form $\Box\left(\mathbb{R}^3\right)$. We can continue this procedure for any arbitrary number of derivatives until we arrive at

我们可以注意到，在式 (31) 中不存在含单个协变导数的项，因为我们可以通过添加任意多个全导数消去所有这类项。例如，假设作用量中有一个形式为 $\mathbb{R}\nabla_\mu\mathbb{R}\nabla^\mu\mathbb{R}$ 的项，那么我们可以通过添加一个形式为 $\Box(\mathbb{R}^3)$ 的全导数消去该项。我们可以对任意数量的导数重复这一过程，最终得到

$$\mathcal{O}_1(\Box_s)\,\mathbb{R}\mathcal{O}_2(\Box_s)\,\mathbb{R}\mathcal{O}_3(\Box_s)\,\mathbb{R}\,, \tag{35}$$

where $\mathbb{R}$ is the curvature quantity. Furthermore, the following nonlocal generalization of Weyl identities [5, 7] is required to write down the cubic nonlocal Weyl tensor terms in (31)

其中 $\mathbb{R}$ 是曲率量。此外，要写出式 (31) 中的三次非局域外尔张量项，还需要用到如下外尔恒等式的非局域推广 [5, 7]

$$W_{[\mu\upsilon}{}^{\gamma\lambda}\mathcal{O}\left(\Box_s\right)W_{\gamma\lambda}{}^{\alpha\beta}\mathcal{O}\left(\Box_s\right)W_{\alpha]\sigma}{}^{\mu\upsilon}=0,$$

$$W^{\mu\upsilon\rho\sigma}\mathcal{O}\left(\Box_s\right)W_{\mu\upsilon\rho\lambda}=\frac{1}{4}\delta^{\sigma}_{\lambda}W^{\mu\upsilon\rho\alpha}\mathcal{O}\left(\Box_s\right)W_{\mu\upsilon\rho\alpha}. \tag{36}$$

In the first line of (36), we have complete antisymmetrization over the five indices. Furthermore, we can in principle make the form factors $\mathcal{C}_i\left(\Box_s\right)$ to depend on R_s , but we simply drop here this generalization for brevity.

式 (36) 第一行中，我们对五个指标做了完全反对称化。此外，原则上我们可以让形状因子 $\mathcal{C}_i\left(\Box_s\right)$ 依赖于 R_s ，但为了简洁，我们在此略去这一推广。

The action (31) must be viewed as leading terms in the series expansion in the approximation $R \ll M_P^2$, and we can expect higher curvature terms that might need to be added to build a full quantum gravity action. Furthermore, we can express the cubic nonlocal form factors as

作用量 (31) 应当被视为 $R \ll M_P^2$ 近似下级数展开的领头项，我们可以预见，构建完整量子引力作用量可能还需要添加更高曲率项。此外，我们可以将三次非局域形状因子表示为

$$\mathcal{L}_i\left(\Box_s\right)=e^{\ell_i\left(\Box_s\right)}-1$$

$$\mathcal{D}_i\left(\Box_s\right)=e^{d_i\left(\Box_s\right)}-1 \tag{37}$$

$$\mathcal{C}_i\left(\Box_s\right)=e^{c_i\left(\Box_s\right)}-1,$$

where $\ell_i\left(\Box_s\right), c_i\left(\Box_s\right), d_i\left(\Box_s\right)$ are the entire functions which we can assume to be polynomials or functions of polynomials that give us a finite parameter space. To have a UV completion, it is expected that the cubic nonlocal operators $\mathcal{L}_i\left(\frac{p^2}{\mathcal{M}_s^2}\right), \mathcal{D}_i\left(\Box_s\right), \mathcal{C}_i\left(\Box_s\right)$ do not grow in the limit $p \to \infty$. This is also a needed behavior in order to have a consistent inflationary predictions as well. Because if these operators are highly suppressed in the $p \to \infty$ limit, all the loop contributions are expected to be subdominant, and the computation of inflationary perturbations at the linear level gives us consistent predictions for correlators.

其中 $\ell_i\left(\Box_s\right), c_i\left(\Box_s\right), d_i\left(\Box_s\right)$ 是整函数，我们可以假定其为多项式或多项式构成的函数，能够给出有限的参数空间。为实现紫外完备，我们要求三阶非局域算符 $\mathcal{L}_i\left(\frac{p^2}{\mathcal{M}_s^2}\right), \mathcal{D}_i\left(\Box_s\right), \mathcal{C}_i\left(\Box_s\right)$ 在 $p \to \infty$ 极限下不发散。这同时也是得到一致暴涨预言的必要条件。因为若这些算符在 $p \to \infty$ 极限下高度压低，所有圈图贡献都将是次要的，线性层面上对暴涨微扰的计算就能给我们给出自洽的关联函数预言。

From (37) and (33), we can write a generic form of entire functions $\gamma_S\left(\Box_s\right)$ and $\ell_i\left(\Box_s\right)$

结合 (37) 和 (33)，我们可以写出整函数 $\gamma_S(\Box_s)$ 与 $\ell_i(\Box_s)$ 的一般形式

$$
\begin{aligned}
\gamma_S(\Box_s) &= \left(\Box_s - \frac{M^2}{\mathcal{M}_s^2}\right) P_S(\Box_s), \\
\ell_i(\Box) &= \left(\Box_s - \frac{M^2}{\mathcal{M}_s^2}\right) G_i(\Box_s), \\
d_1(\Box) &= \left(\Box_s - \frac{M^2}{\mathcal{M}_s^2}\right) D_1(\Box_s),
\end{aligned}
\tag{38}
$$

where $P_S(\Box_s), G_i(\Box_s), D_1(\Box_s)$ are the finite degree polynomials.

其中 $P_S(\Box_s), G_i(\Box_s), D_1(\Box_s)$ 是有限次多项式。

Before going to the next section where we discuss inflationary observables of R^2 -like inflation in GNLQG (31), let us make here an intuitive remark that the presence of nonlocal term $RW_{\mu\upsilon\rho\sigma}\mathcal{F}_W(\Box_s, R_s)W^{\mu\upsilon\rho\sigma}$ does not necessarily spoil the power counting renormalizability which was achieved in NLQG [16]. We can easily verify this by taking the generalized ghost-free form factor (32) of the action (31) in the high energy limit, and the N -point graviton vertices arising from $W_{\mu\upsilon\rho\sigma}\mathcal{O}(\Box_s)W^{\mu\nu\rho\sigma}$ dominate over the N -point vertices arising from $R^{N-2}W_{\mu\upsilon\rho\sigma}W^{\mu\upsilon\rho\sigma}$. Therefore, the traditional power counting renormalizability around Minkowski background which is studied in [30] most likely holds for GNLQG (31), especially if we arrange cubic nonlocal form factors such that they are highly suppressed in the high energy limit. However, still renormalizability might require more additional terms to those present in (31) that is an open problem and requires further investigation.

在进入下一节讨论 GNLQG(31) 中 R^2 型暴涨的暴涨可观测物理量之前，我们先给出一个直观的说明: 非局域项 $RW_{\mu\upsilon\rho\sigma}\mathcal{F}_W(\Box_s, R_s)W^{\mu\upsilon\rho\sigma}$ 的存在并不一定会破坏 NLQG 中已经实现的幂次计数可重整性 [16]。我们可以很容易验证这一点: 对作用量 (31) 的推广无鬼形状因子 (32) 取高能极限，由 $W_{\mu\upsilon\rho\sigma}\mathcal{O}(\Box_s)W^{\mu\nu\rho\sigma}$ 产生的 N 点引力子顶点会主导由 $R^{N-2}W_{\mu\upsilon\rho\sigma}W^{\mu\upsilon\rho\sigma}$ 产生的 N 点顶点。因此，[30] 中研究的闵氏背景下的传统幂次计数可重整性极有可能对 GNLQG(31) 成立，如果我们调整三阶非局域形状因子，使其在高能极限下高度压低，这一点就更成立了。不过，可重整性或许仍然要求 (31) 中现有项之外补充更多额外项；这是一个开放问题，有待进一步研究。

Predictions of Generalized Nonlocal R^2 -Like Inflation

广义非局部 R^2 型暴胀的预言

In the previous section, we established R^2 -like inflation in GNLQG, and in this section we review the predictions of the model with the 2-point and 3-point inflationary correlations. These results are derived in [32, 33] which we briefly review here.

在上一节中，我们在广义非局部二次引力中建立了 R^2 型暴胀，本节我们将利用二阶与三阶暴胀关联回顾该模型的预言。这些结果已在文献 [32, 33] 中推导得出，本文在此做简要回顾。

The second-order perturbation of the action (31) around the inflationary solution that follows from $\bar{\Box}\bar{R} = M^2\bar{R}$ can be calculated as

可以计算由 $\bar{\Box}\bar{R} = M^2\bar{R}$ 得到的暴胀解附近作用量 (31) 的二阶微扰如下:

$$\delta^{(2)} S_H^{\text{Nonlocal}} = \delta^{(2)} S_{R+R^2}^{\text{local}} + \delta^{(2)} S_{R+R^2}^{\text{Nonlocal}} + \delta^{(2)} S_{\mathbb{R}^3}^{\text{Nonlocal}}, \tag{39}$$

where $S_{R+R^2}^{\text{local}}$ is the local R^2 action and

其中 $S_{R+R^2}^{\text{local}}$ 是局部 R^2 作用量，且

$$\begin{aligned} S_{R+R^2}^{\text{Nonlocal}} &= S_{R^2}^{\text{Nonlocal}} + S_{W^2}^{\text{Nonlocal}} \\ &= \frac{1}{2}\int d^4x\sqrt{-g}\{R\left[\mathcal{F}_R(\Box_s) - f_0\right]R \\ &+ \left(\frac{M_p^2}{2\mathcal{M}_s^2} + f_0\frac{R}{\mathcal{M}_s^2}\right)W_{\mu\nu\rho\sigma}\mathcal{F}_W\left(\Box_s, \frac{R}{\mathcal{M}_s^2}\right)W^{\mu\nu\rho\sigma}\right\} \end{aligned} \tag{40}$$

and $S_{\mathbb{R}^3}^{\text{Nonlocal}}$ constitute from the last three lines in (31) which are cubic in curvature. Applying the conditions (33), we can see that the cubic nonlocal term has no contribution to the above second-order action. This is not very surprising, and we can understand this following an analogous simple example given by the action:

而 $S_{\mathbb{R}^3}^{\text{Nonlocal}}$ 由 (31) 中最后三行构成，这些项是曲率的三次项。应用条件 (33) 可知，三次非局部项对上述二阶作用量没有贡献。这并不出人意料，我们可以通过如下作用量给出的类似简单例子理解这一点:

$$S_{R^3}^{\text{Min}} = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_p^2 R + f_0 R^2 + f_0\Lambda_m^{-2}R^3\right], \tag{41}$$

where Λ_m is some mass scale. From the above we can notice that the cubic term R^3 does not affect the second-order action (or the linearized equations of motion) around Minkowski; however, it is obviously not true in other backgrounds. If we study inflation with local higher curvature extension of R^2 , we do modify inflationary solution unless we assume R^2 is the most relevant term during inflation. In fact, slow-roll inflation in local $f(R)$ gravity occurs for the range of R for which the Lagrangian density $\mathcal{L}$ is close to R^2 , namely, $\mathcal{L} = A(R)R^2$, where $A(R)$ is a slowly changing function of $R, \left|\frac{d\ln A}{d\ln R}\right| \ll 1$ [90]. In our case the cubic scalar curvature nonlocal terms (i.e., the last three lines in (31)) are introduced in such a way that we do not get any contributions to the 2-point correlations which means

其中 Λ_m 是某个质量标度。从上式我们可以看出，三次项 R^3 不影响闵氏背景附近的二阶作用量 (或线性化运动方程)；但在其他背景下情况显然并非如此。如果我们研究带有 R^2 局部高曲率推广的暴胀，除非假设 R^2 是暴胀期间最相关的项，否则我们确实会改变暴胀解。实际上，局部 $f(R)$ 引力中的慢滚暴胀发生在拉格朗日密度 $\mathcal{L}$ 接近 R^2 的 R 范围内，即满足 $\mathcal{L} = A(R)R^2$ ，其中 $A(R)$ 是 $R, \left|\frac{d\ln A}{d\ln R}\right| \ll 1$ 的慢变函数 [90]。在本文的情形中，三次标曲率非局部项 (即 (31) 的最后三行) 的引入方式使得它们对两点关联没有任何贡献，这意味着

$$\delta^{(2)} S_{\mathbb{R}^3}\big|_{\Box \bar{R}=M^2\bar{R}} = 0. \tag{42}$$

However, the cubic nonlocal terms are expected to contribute to the third-order perturbation of the action (31) which implies the last three lines in (31) affect the primordial non-Gaussianities which we discuss in the next subsection. After long computations, we deduce that second-order action in (39) exactly coincides with the result obtained in the context of NLQG [31], which is not surprising because our construction of GNLQG is an extension of NLQG preserving the already interesting inflationary predictions of R^2 -like inflation in NLQG [19,30,31]. The crucial difference in GNLQG is that we consider a choice of $\mathcal{F}_R(\Box_s), \mathcal{F}_W(\Box_s)$ that does not explicitly depend on the background curvatures during inflation which was the case of earlier NLQG frameworks [19,29,31]. This ascertains that GNLQG is more fundamental action and is more toward the full description of quantum gravity in the nonlocal setup.

但三次非局部项预计会对作用量 (31) 的三阶微扰有贡献，也就是说 (31) 的最后三行会影响原初非高斯性，我们将在下一小节讨论这点。经过大量计算，我们推导出 (39) 中的二阶作用量与非局部量子引力 (NLQG) 框架下得到的结果完全一致 [31]，这并不出人意料，因为我们的广义非局部二次引力 (GNLQG) 构造是对 NLQG 的推广，保留了 NLQG 中 R^2 型暴胀已有的有趣暴胀预言 [19,30,31]。GNLQG 的关键区别在于，我们选取的 $\mathcal{F}_R(\Box_s), \mathcal{F}_W(\Box_s)$ 不显式依赖暴胀期间的背景曲率，而早期 NLQG 框架 [19,29,31] 并非如此。这表明 GNLQG 是更基本的作用量，更接近非局部框架下量子引力的完整描述。

To compute the scalar power spectrum, we start with the second-order action for the scalar perturbation given in (23) which happened to be the same both in NLQG and in GNLQG. We use the form factor (32) for which we have seen from (26) that there is only one scalaron, and we set the initial conditions of all the modes corresponding to complex conjugate poles. Thus the calculation of scalar power spectrum $\mathcal{P}_{\mathcal{R}}$ and the spectral index n_s of R^2 -like inflation in GNLQG are pretty much standard by the use of canonical rescaling of the fields, and the final results are [32]

为了计算标量功率谱，我们从式 (23) 给出的标量微扰二阶作用量出发，该作用量在非局部量子引力和广义非局部量子引力中恰好形式相同。我们采用式 (32) 的形状因子，从式 (26) 可知该形状因子仅对应一个标量子，我们对所有对应复共轭极点的模式设定初始条件。因此，通过场的正则标度变换，广义非局部量子引力中 R^2 型暴涨的标量功率谱 $\mathcal{P}_{\mathcal{R}}$ 和谱指数 n_s 的计算十分标准，最终结果见文献 [32]

$$\mathcal{P}_{\mathcal{R}}(k) \approx \frac{1}{3f_0\bar{R}_{\rm dS}} \frac{H^2}{16\pi^2\varepsilon^2}\bigg|_{k=aH}, \quad n_s - 1 \equiv \frac{d\ln \mathcal{P}_{\mathcal{R}}}{d\ln k}\bigg|_{k=aH} \approx -\frac{2}{N}, \tag{43}$$

where $a(t)H(t)$ is estimated at the first Hubble radius crossing during inflation for a given mode wave number k . The above result (43) is almost independent of $\mathcal{F}_R(\Box_s)$ as far as $M^2 \ll \mathcal{M}_s^2$, and the dimensionless coefficients of $\mathcal{F}_R(\Box_s)$ (32) are of the $O(1)$. Moreover, the scalar slope n_s is practically the same as in the Starobinsky inflationary model (2) [91,92], see [19,31] for further details where it is shown that the nonlocal correction to it is of the order of $O\left(\frac{M^4}{\mathcal{M}_s^4}\right)$.

其中 $a(t)H(t)$ 是暴胀期间，对给定波数 k 的模式在哈勃半径第一次穿越时的估计值。只要满足 $M^2 \ll \mathcal{M}_s^2$，上述结果 (43) 几乎与 $\mathcal{F}_R(\Box_s)$ 无关，且 (32) 中 $\mathcal{F}_R(\Box_s)$ 的无量纲系数属于 $O(1)$。此外，标量谱斜率 n_s 实际上与斯塔罗宾斯基暴胀模型 (2)[91,92] 完全一致，进一步细节参见文献 [19,31]，其中表明对它的非局部修正量级为 $O\left(\frac{M^4}{\mathcal{M}_s^4}\right)$。

In the inflationary high-curvature regime $R \gg M^2$, quadratic curvature terms are naturally dominant. Thus, the hierarchy $M^2 \ll \mathcal{M}_s^2$ is essential to have the generalized nonlocal R^2 -like inflation to be compatible with Planck data. On the other hand, H^2 can be of the order or even larger than $\mathcal{M}_s^2$ while still being much less than M_p^2; see the hierarchy of scales in Fig. 1. This can be seen heuristically by expanding the quadratic Ricci scalar part of the action (31) as

在暴胀的高曲率区域 $R \gg M^2$，二次曲率项自然占主导。因此，层级关系 $M^2 \ll \mathcal{M}_s^2$ 对保证广义非局部 R^2 类暴胀与普朗克观测数据相容是必要的。另一方面，H^2 的量级可以等于甚至大于 $\mathcal{M}_s^2$，同时仍远小于 M_p^2；标度的层级关系参见图 1。这可以通过对作用量 (31) 中的二次里奇标量部分做展开得到直观说明:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + \frac{M_p^2}{12M^2}R^2 + O\left(\frac{M_p^2 R\Box R}{M^2\mathcal{M}_s^2}\right) + \cdots\right]. \tag{44}$$

Fig. 1 The hierarchy of in the generalized nonlocal R^2 -like inflation

图 1 广义非局部 R^2 类暴胀中的层级关系

In order to compute the tensor power spectrum, we use the second-order action for the tensor mode (21) which turned out to be exactly same in both NLQG and GNLQG [32]. To compute tensor power spectrum, we do the rescaling of $h_{ij} \to e^{\gamma_T\left(\Box_{\rm dS} - \frac{\bar{R}_{\rm dS}}{3\mathcal{M}_s^2}\right)} h_{ij}$ and bring the action (21) into the local form. Then, after performing the canonical quantization of the field by standard methods, we will rescale back the result by evaluating the exponent of entire function at the pole $\overline{\Box}_{\rm dS} = \frac{\bar{R}_{\rm dS}}{6}$ [30, 32]. As a result, we get the inflationary tensor power spectrum of generalized nonlocal R^2 model as

为了计算张量功率谱，我们使用张量模式的二阶作用量 (21)，该作用量在 NLQG 和 GNLQG 中结果完全一致 [32]。为计算张量功率谱，我们对 $h_{ij} \to e^{\gamma_T\left(\Box_{\rm dS} - \frac{\bar{R}_{\rm dS}}{3\mathcal{M}_s^2}\right)} h_{ij}$ 做重标度变换，将作用量 (21) 化为局域形式。随后，用标准方法对场做正则量子化后，我们会通过计算整函数在极点 $\overline{\Box}_{\rm dS} = \frac{\bar{R}_{\rm dS}}{6}$ [30, 32] 处的指数，对结果做逆重标度变换。最终我们得到广义非局部 R^2 模型的暴胀张量功率谱为:

$$\mathcal{P}_T = \frac{1}{12\pi^2 f_0}(1-3\varepsilon)\, e^{-2\gamma_T\left(\frac{-\bar{R}_{dS}}{6\mathcal{M}_s^2}\right)}\Bigg|_{k=aH}. \tag{45}$$

The tensor-to-scalar ratio from (43) and (45) can be calculated as

由 (43) 和 (45)，张标比可计算为

$$r = \frac{12}{N^2} e^{-2\gamma_T\left(-\frac{\bar{R}_{\rm dS}}{6\mathcal{M}_s^2}\right)}\Big|_{k=aH}. \tag{46}$$

The tensor spectral index and its running and running of running can be evaluated as

张量谱指数、它的跑动和跑动的跑动可计算为

$$n_t \equiv \frac{d\ln\mathcal{P}_T}{d\ln k}\Big|_{k=aH} \approx -\frac{3}{2N^2} - \left(\frac{2}{N} + \frac{1}{N^2}\right)\frac{\bar{R}_{\rm dS}}{6\mathcal{M}_s^2}\gamma_T^{\dagger}\left(-\frac{\bar{R}_{\rm dS}}{6\mathcal{M}_s^2}\right)$$

$$\frac{dn_t}{d\ln k}\Big|_{k=aH} \approx -\frac{3}{N^3} - \frac{1}{N^3}\frac{\bar{R}_{\rm dS}}{6\mathcal{M}_s^2}\gamma_T^{\dagger}\left(-\frac{\bar{R}_{\rm dS}}{6\mathcal{M}_s^2}\right) - \frac{1}{18N^2}\frac{\bar{R}_{\rm dS}^2}{\mathcal{M}_s^4}\gamma_T^{\dagger\dagger}\left(-\frac{\bar{R}_{\rm dS}}{6\mathcal{M}_s^2}\right)$$

$$\frac{d^2n_t}{d\ln k^2}\Big|_{k=aH} \approx -\frac{9}{N^4} - \frac{1}{3N^4}\frac{\bar{R}_{\rm dS}}{\mathcal{M}_s^2}\gamma_T^{\dagger}\left(-\frac{\bar{R}_{\rm dS}}{6\mathcal{M}_s^2}\right) - \frac{1}{12N^3}\frac{\bar{R}_{\rm dS}^2}{\mathcal{M}_s^4}\gamma_T^{\dagger\dagger}\left(-\frac{\bar{R}_{\rm dS}}{6\mathcal{M}_s^2}\right)$$

$$-\frac{1}{108N^3}\frac{\bar{R}_{\rm dS}^3}{\mathcal{M}_s^6}\gamma_T^{\dagger\dagger\dagger}\left(-\frac{\bar{R}_{\rm dS}}{6\mathcal{M}_s^2}\right),$$

(47)

where †, ††, ††† , ††† indicate the first, second, and third derivatives with respect to the argument, and we used $\frac{d}{d\ln k} \approx -\left(1+\frac{1}{2N}\right)\frac{d}{dN}$ and $\frac{d\bar{R}_{\rm dS}}{dN} \approx 2\bar{R}_{\rm dS}\varepsilon$. From (45), (46), and (47), we can conclude the following:

其中 †, ††, ††† 、††† 分别表示对宗量的一阶、二阶和三阶导数，我们使用了 $\frac{d}{d\ln k} \approx -\left(1+\frac{1}{2N}\right)\frac{d}{dN}$ 和 $\frac{d\bar{R}_{\rm dS}}{dN} \approx 2\bar{R}_{\rm dS}\varepsilon$ 。由式 (45)、(46) 和 (47)，我们可以得到以下结论:

- The tensor power spectrum is modified due to the higher curvature nonlocal terms involving Ricci scalar and Weyl tensor in (31).

- 由于 (31) 中包含里奇标量和外尔张量的高阶曲率非局部项，张量功率谱发生了修正。

- Compared to the result in the local $R+R^2$ gravity (2) [92], the tensor power spectrum contains a strong scale dependence due to the exponential term $e^{-2\gamma_T\left(\frac{-\bar{R}_{\rm dS}}{6\mathcal{M}_s^2}\right)}$, where $\bar{R}_{\rm dS}(k)$ depends on wave number k .

- 与局域 $R+R^2$ 引力 (式 (2)，文献 [92]) 的结果相比，由于指数项 $e^{-2\gamma_T\left(\frac{-\bar{R}_{\rm dS}}{6\mathcal{M}_s^2}\right)}$ ，张量功率谱存在强标度依赖性，其中 $\bar{R}_{\rm dS}(k)$ 依赖于波数 k 。

- With tensor tilt and its running and running of running computed in from (47), we can probe $\gamma_T\left(-\frac{\bar{R}_{\rm dS}}{6\mathcal{M}_s^2}\right)$ and reconstruct the form factor (32) from the future primordial gravitational wave observations [47, 93] .

- 利用由式 (47) 计算得到的张量倾斜量、其一阶跑动和二阶跑动，我们可以通过未来原初引力波观测 [47, 93] 探测 $\gamma_T\left(-\frac{\bar{R}_{\rm dS}}{6\mathcal{M}_s^2}\right)$ 并重构形状因子 (32)。

- The single-field tensor consistency relation $r = -8n_t$ gets violated solely due to the modification of tensor-power spectrum.

> - 单场张量一致性关系 $r = -8n_t$ 仅因张量功率谱的修正而被破坏。

In Fig. 2 we depict the status of R^2 -like inflation in GNLQG against the latest constraints from Planck+BICEP/Keck analysis where we can see that in this model we can have any tensor-to-scalar ratio $r < 0.036$. This has been shown explicitly by the simplest choices of the entire function [32]

> 在图 2 中，我们给出了广义非局部量子引力中 R^2 类暴胀面对 Planck+BICEP/Keck 最新观测限制的情况，可以看到该模型中张量标量比 $r < 0.036$ 可以取任意值。这一点已经通过对整函数的最简单选取被明确验证 [32]

$$\gamma_T\left(\Box_s - \frac{2R_s}{3}\right) = \beta_1\left(\Box_s - \frac{2R_s}{3}\right)^2 + \beta_2\left(\Box_s - \frac{2R_s}{3}\right)^3 + \cdots, \tag{48}$$

where $\cdots$ denotes the possible higher order terms that can be irrelevant in the context of inflation, but such terms could play a role in taming UV divergences. In the next section, we shall come back to these predictions and compare them with the EFT models of inflation.

> 其中 $\cdots$ 代表可能的高阶项，这些项在暴胀背景下无关紧要，但却能够帮助抑制紫外发散。下一节我们将回到这些预言，将它们与暴胀的有效场论模型进行比较。

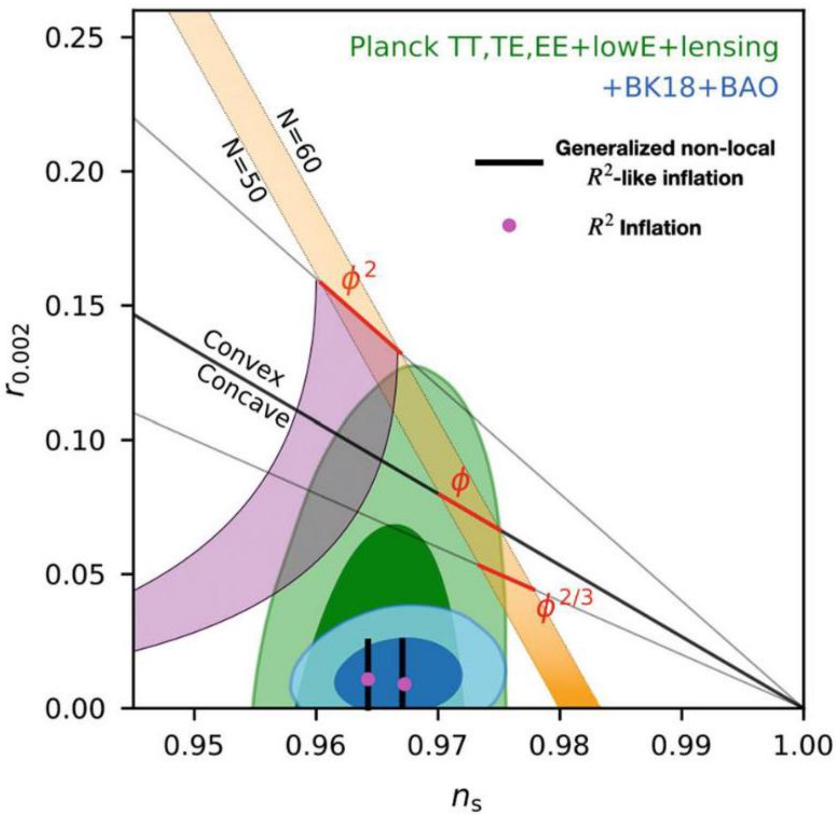


Fig. 2 Here we depict the predictions of generalized nonlocal R^2 -like inflation with respect to the Planck and BICEP/Keck array analysis (with the ruled out monomial models of inflation and the natural inflation

shown in purple color) which has constrained the tensor-to-scalar ratio as $r < 0.036$ and the spectral index as $n_s = 0.9649 \pm 0.0042$. The black color vertical line from the left represents $(n_s, r) = (0.964, < 0.036)$ for $N = 55$, whereas the one on the right represents $(n_s, r) = (0.967, < 0.036)$ for $N = 60$

图 2 我们在此给出广义非局部 R^2 类暴胀相对于 Planck 与 BICEP/Keck 阵列分析的预言 (已排除的单项式暴胀模型和自然暴胀模型以紫色标出)，该分析将张量标量比限制为 $r < 0.036$ ，谱指数限制为 $n_s = 0.9649 \pm 0.0042$ 。左侧黑色竖线代表对应 $N = 55$ 的 $(n_s, r) = (0.964, < 0.036)$ ，右侧黑色竖线代表对应 $N = 60$ 的 $(n_s, r) = (0.967, < 0.036)$

In Fig. 3 we depict the fact that the generalized nonlocal R^2 -like inflation can predict both positive and negative values of tensor tilt within the likelihood region from the Planck data.

在图 3 中我们表明，广义非局部 R^2 类暴胀在 Planck 数据的似然区域内既可以预言正的张量倾斜量，也可以预言负的张量倾斜量。

Primordial (Scalar) Non-Gaussianities

原初 (标量) 非高斯性

PNGs are of enormous importance in the R^2 -like inflation in GNLQG, because when we calculate interaction vertices in nonlocal gravity, we most often get the presence of AID operators unlike the local models of inflation [94]. Therefore, PNGs are the most pertinent observables to find the signature of nonlocalities. In this section we review only the scalar PNGs in GNLQG which are derived in [33].

原初非高斯性 (PNGs) 在广义非局部二次引力 (GNLQG) 的 R^2 型暴胀中至关重要，因为在非局部引力中计算相互作用顶点时，与局域暴胀模型不同，我们通常会得到 AID 算符 [94]。因此，原初非高斯性是寻找非局域特征最相关的可观测量。本节仅综述文献 [33] 中推导的 GNLQG 中的标量原初非高斯性。

PNGs are calculated with computing 3-point correlations defined by Maldacena [95] and Koshelev et al. [19]

原初非高斯性通过计算由 Maldacena[95] 和 Koshelev 等人 [19] 定义的三点关联函数得到

$$\langle \mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3)\rangle = -i\int_{-\infty}^{\tau_e} d\tau \langle 0 | [\mathcal{R}(\tau_e,\mathbf{k}_1)\mathcal{R}(\tau_e,\mathbf{k}_2)\mathcal{R}(\tau_e,\mathbf{k}_3), H_{\text{int}}] | 0\rangle ,$$

(49)where $\mathbf{k}_i$ are wave vectors, $H_{\text{int}} \approx -\mathcal{L}_3$ is the interaction Hamiltonian that is approximately equal to the third-order perturbation of the Lagrangian (31) ($\mathcal{L}_3$) within the slow-roll approximation [95, 96] , and τ_e denotes the end of inflation.

其中 $\mathbf{k}_i$ 是波矢，$H_{\text{int}} \approx -\mathcal{L}_3$ 是相互作用哈密顿量，在慢滚近似 [95, 96] 下它近似等于拉格朗日量 (31) ($\mathcal{L}_3$) 的三阶微扰，τ_e 表示暴胀结束时刻。

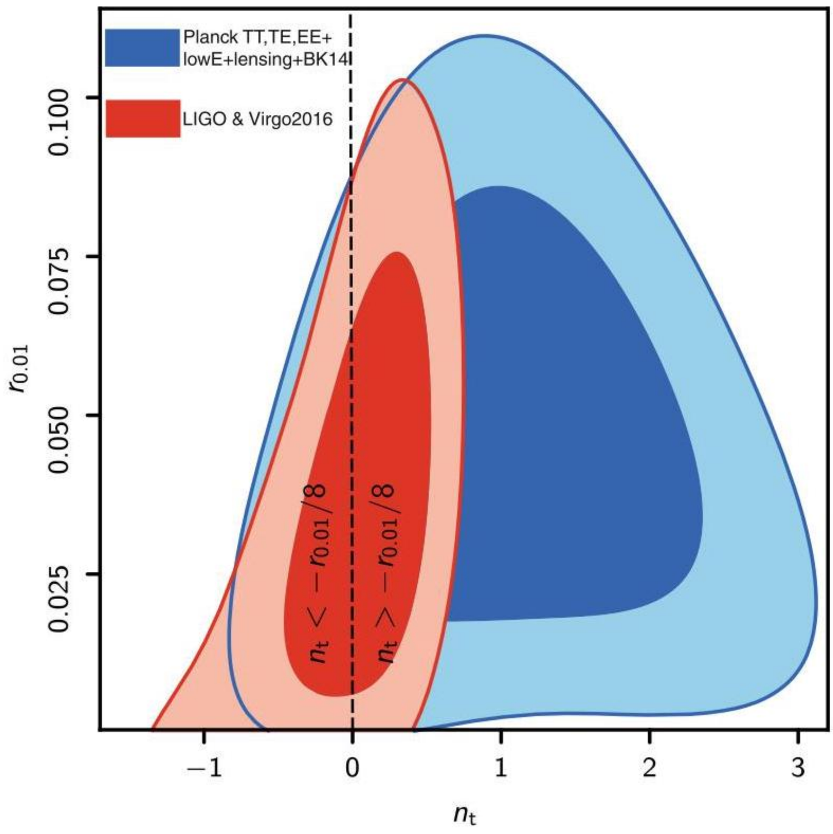


Fig. 3 This figure represents $n_t - r$ plane from Planck 2018 data. The predictions of generalized nonlocal R^2 -like inflation can be anywhere within the plane. If future experiments detect actual values of these quantities, we shall be able to constrain more precisely the form factors and the scale of nonlocality

图 3 本图展示了普朗克 2018 数据中的 $n_t - r$ 平面。广义非局部 R^2 型暴胀的预言可以落在平面内任意位置。如果未来实验探测到这些量的实际值，我们就能更精确地约束形状因子和非局域能标。

The bispectrum ($\mathcal{B}_{\mathcal{R}}$) is usually defined as

双谱 ($\mathcal{B}_{\mathcal{R}}$) 通常定义为

$$\langle \mathcal{R}(\mathbf{k_1})\mathcal{R}(\mathbf{k_2})\mathcal{R}(\mathbf{k_3})\rangle = (2\pi)^3 \delta^3(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\mathcal{B}_{\mathcal{R}}(k_1,k_2,k_3), \tag{50}$$

where $|\mathbf{k}_i| = k_i$, and the nonlinear curvature perturbation $\mathcal{R}$ is expressed as [97, 98]

其中 $|\mathbf{k}_i| = k_i$ ， 非线性曲率扰动 $\mathcal{R}$ 可表示为 [97, 98]

$$\mathcal{R} = \mathcal{R}_g - \frac{3}{5} f_{NL}\left(\mathcal{R}_g^2 - \langle \mathcal{R}_g \rangle^2\right), \tag{51}$$

where $\mathcal{R}_g$ is the Gaussian random field and the f_{NL} is the nonlinearity parameter also known as the reduced bispectrum [99] that is defined as

其中 $\mathcal{R}_g$ 是高斯随机场，f_{NL} 是非线性参数，也被称为约化双谱 [99]，其定义为

$$f_{NL} = -\frac{5}{6}\frac{A_{\mathcal{R}}(k_1,k_2,k_3)}{\sum\limits_i k_i^3}, \tag{52}$$

where $A_{\mathcal{R}}(k_1,k_2,k_3)$ stands for the redefinition of the bispectrum $B_{\mathcal{R}}$:

其中 $A_{\mathcal{R}}(k_1,k_2,k_3)$ 代表对双谱 $B_{\mathcal{R}}$ 的重新定义:

$$B_{\mathcal{R}}(k_1,k_2,k_3) = 4\pi^4\frac{1}{\prod\limits_i k_i^3}\mathcal{P}_{\mathcal{R}}^2 A_{\mathcal{R}}(k_1,k_2,k_3). \tag{53}$$

To calculate f_{NL} , we calculate the third-order variation of (31) around the background (16) as

为计算 f_{NL} ，我们对背景 (16) 附近的 (31) 计算三阶变分，得到

$$\delta_{(s)}^{(3)}S_H^{\text{Nonlocal}} = \delta_{(s)}^{(3)}S_{R+R^2}^{\text{local}} + \delta_{(s)}^{(3)}S_{R+R^2}^{\text{Nonlocal}} + \delta_{(s)}^{(3)}S_{\mathbb{R}^3}^{\text{Nonlocal}}, \tag{54}$$

where

其中

$$\begin{aligned}
S_{R+R^2}^{\text{local}} &= \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + \frac{f_0}{2}R^2\right] \\
S_{R+R^2}^{\text{Nonlocal}} &= \int d^4x\sqrt{-g}\left\{\frac{M_p^2}{2}R + \frac{1}{2}R\left[\mathcal{F}_R(\Box_s) - f_0\right]R\right\} \\
S_{R^3}^{\text{Nonlocal}} &= \int d^4x\sqrt{-g}\left[\mathcal{L}_1(\Box_s)R\mathcal{L}_2(\Box_s)R\mathcal{L}_3(\Box_s)R\right],
\end{aligned} \tag{55}$$

and the subscript $_{(s)}$ in (54) denotes the scalar part of the third-order action as we are not discussing tensor PNGs here. Using the calculations performed in [19], since $\Phi + \Psi \approx 0$ during inflation and the variation $\delta_{(s)}W_{\mu\upsilon\rho\sigma} \propto \Phi + \Psi$, we can conclude that all the terms involving Weyl tensor do not contribute to the scalar PNGs. In (31), we can further possibly consider quartic order nonlocal scalar curvature term:

(54) 的下标 $_{(s)}$ 表示三阶作用量的标量部分，因为本文不讨论张量原初非高斯性。利用文献 [19] 的计算，由于暴胀期间 $\Phi+\Psi\approx 0$ 且变分 $\delta_{(s)}W_{\mu\upsilon\rho\sigma} \propto \Phi+\Psi$，我们可以得到结论: 所有包含外尔张量的项对标量原初非高斯性都没有贡献。在 (31) 中，我们还可以进一步考虑四次阶非局部标量曲率项:

$$S_{R^4}^{\text{Nonlocal}} = \frac{f_0\lambda_q}{2\mathcal{M}_s^4}\int d^4x\sqrt{-g}\left[\mathcal{L}_4(\Box_s)R\mathcal{L}_1(\Box_s)R\mathcal{L}_2(\Box_s)R\mathcal{L}_3(\Box_s)R\right]. \tag{56}$$

Here $\mathcal{L}_4(\Box_s)$ is an arbitrary analytic infinite derivative operator. It is easy to deduce that (56) still admits the inflationary solution (16). Applying $\mathcal{L}_i\left(\frac{M^2}{\mathcal{M}_s^2}\right) = 0 \ \ (i = 1,2,3)$, we can conclude that the second-order variation of (56) around the background satisfying (16) becomes zero exactly

此处 $\mathcal{L}_4(\Box_s)$ 是任意解析无穷导数算符。不难推导出 (56) 仍然容许暴胀解 (16)。应用 $\mathcal{L}_i\left(\frac{M^2}{\mathcal{M}_s^2}\right) = 0$ $(i = 1,2,3)$，我们可以得到结论: 满足 (16) 的背景附近，(56) 的二阶变分恰好为零

$$\delta^{(2)} S_{R^4}^{\text{Nonlocal}} = 0, \tag{57}$$

whereas the third-order variation of (56) around (16) in the leading order de Sitter approximation is

而在领头阶德西特近似下，(16) 附近 (56) 的三阶变分为

$$\delta^{(3)} S_{R^4}^{\text{Nonlocal}} \approx \mathcal{L}_4\left(\frac{M^2}{\mathcal{M}_s^2}\right)\frac{\lambda_q \bar{R}_{\text{dS}}}{\lambda_c \mathcal{M}_s^2}\delta^{(3)} S_{R^3}^{\text{Nonlocal}}. \tag{58}$$

Given that

考虑到

$$\mathcal{L}_4\left(\frac{M^2}{\mathcal{M}_s^2}\right)\frac{\lambda_q \bar{R}_{\text{dS}}}{\lambda_c \mathcal{M}_s^2} \ll 1, \tag{59}$$

we can neglect (56) for the scalar (3-point) PNGs. However, we can expect to have nonnegligible contributions to the trispectrum (or) 4-point correlations due to this term. The same logic can be easily extended to other higher order scalar curvature nonlocal terms and for the higher order Weyl curvature nonlocal terms.

我们可以忽略 (56) 对标量 (三点) 原初非高斯性的贡献。但我们可以预期该项对三频谱 (即四点关联函数) 会有不可忽略的贡献。这套逻辑可以很容易推广到其他高阶标量曲率非局部项和高阶外尔曲率非局部项。

Therefore, we can conclude that nonlocal contributions to the bispectrum arise only from the part of the action (31) which is quadratic and cubic in Ricci scalar. After the long computations, we can rewrite (54) in terms of curvature perturbation $\mathcal{R}$ as

因此，我们可以得出结论: 双谱的非局域贡献仅来自作用量 (31) 中里奇标量为二次和三次的部分。经过冗长计算，我们可将 (54) 用曲率扰动 $\mathcal{R}$ 改写为

$$\delta_{(s)}^{(3)} S_H^{\text{Nonlocal}}$$

$$= 4\frac{M_p^2}{H^2}\varepsilon \int d\tau d^3x \left\{B_1\tau^{-2}\mathcal{R}\nabla\mathcal{R}\cdot\nabla\mathcal{R} + B_2\tau^{-2}\mathcal{R}\mathcal{R}'^2 + B_3\tau^{-3}\mathcal{R}\mathcal{R}\mathcal{R}'\right.$$

$$\left.+B_4\tau^{-1}\mathcal{R}'^3 + B_5\tau^{-4}\mathcal{R}^3 + B_6\tau^{-1}\nabla\mathcal{R}\cdot\nabla\mathcal{R}\mathcal{R}' + B_7\mathcal{R}'\nabla\mathcal{R}\cdot\nabla\mathcal{R}'\right\},$$

(60)where $B_1 - B_7$ are dimensionless parameters approximated to be constant during inflation which are given by

其中 $B_1 - B_7$ 是暴胀期间近似为常数的无量纲参数，由下式给出

$$B_1 = -2\varepsilon - \frac{3\varepsilon^2}{4}$$

$$B_2 = 2\varepsilon + \frac{3\varepsilon^2}{4} + \frac{16}{3}\varepsilon\mathcal{T}_{\rm NL} + \frac{8}{3}\varepsilon^3\frac{\bar{R}_{\rm dS}^2}{\mathcal{M}_S^4}\gamma_S^\dagger\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)e^{\gamma_S\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)}$$

$$-\frac{2\lambda_c}{9}\frac{\bar{R}_{\rm dS}}{\mathcal{M}_s^2}\varepsilon\left(2\varepsilon^2T_{\rm NL}^3 + \varepsilon T_{\rm NL}^2 + T_{\rm NL}^1\right)$$

$$B_3 = -32\mathcal{T}_{\rm NL} - \frac{8\lambda_c}{9}\frac{\bar{R}_{\rm dS}}{\mathcal{M}_S^2}\varepsilon\left(2\varepsilon T_{\rm NL}^2 + T_{\rm NL}^1\right) \tag{61}$$

$$B_4 = -2\mathcal{T}_{\rm NL} - \frac{1}{54}\frac{\bar{R}_{\rm dS}}{\mathcal{M}_s^2}\varepsilon\left(8\varepsilon^3T_{\rm NL}^4 + 4\varepsilon^2T_{\rm NL}^3 + 2\varepsilon T_{\rm NL}^2 + T_{\rm NL}^1\right)$$

$$B_5 = -\frac{\varepsilon^2}{2} + \frac{32\bar{R}_{\rm dS}}{\mathcal{M}_s^2}\varepsilon T_{\rm NL}^1$$

$$B_6 = -2\mathcal{T}_{\rm NL}$$

$$B_7 = \frac{16}{3}\varepsilon\mathcal{T}_{\rm NL} + \frac{8}{3}\varepsilon^3\frac{\bar{R}_{\rm dS}^2}{\mathcal{M}_S^4}\gamma_S^\dagger\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)e^{\gamma_S\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)},$$

where the subscript " ^4S " denotes the quantities in quasi-dS approximation. And the quantities $\mathcal{T}_{\rm NL}, T_{\rm NL}^1, \cdots, T_{\rm NL}^4$ are given by

其中下标" ^4S " 表示准德西特近似下的物理量，物理量 $\mathcal{T}_{\rm NL}, T_{\rm NL}^1, \cdots, T_{\rm NL}^4$ 由下式给出

$$\mathcal{T}_{\rm NL} = \frac{\bar{R}_{\rm dS}}{M_p^2}\varepsilon^3\left[\mathcal{F}_R\left(\frac{M^2}{\mathcal{M}_s^2} + \frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right) - \mathcal{F}_1\right]$$

$$\approx \frac{\bar{R}_{\rm dS}}{M_p^2}\varepsilon^3\left[\mathcal{F}_R\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right) - \mathcal{F}_1 + \varepsilon\frac{\bar{R}_{\rm dS}}{8\mathcal{M}_s^2}\mathcal{F}_R^{(\dagger)}\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)\right]$$

$$\approx \frac{1}{3}\varepsilon^2\left(e^{\gamma_S\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)} - 1\right) + \varepsilon^3\frac{\bar{R}_{\rm dS}^2}{12\mathcal{M}_S^4}\gamma_S^\dagger\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)e^{\gamma_S\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)}$$

$$T_{\rm NL}^1 = \sum_{i,j,k,i\neq j\neq k}\mathcal{L}_i\left(\frac{2M^2}{\mathcal{M}_s^2}\right)\mathcal{L}_j\left(\frac{2M^2}{\mathcal{M}_s^2}\right)\mathcal{L}_k\left(\frac{2M^2}{\mathcal{M}_s^2}\right),$$

$$T_{\rm NL}^2 = \sum_{i,j,k,i\neq j\neq k}\mathcal{L}_i\left(\frac{2M^2}{\mathcal{M}_s^2}\right)\mathcal{L}_j\left(\frac{2M^2}{\mathcal{M}_s^2}\right)\mathcal{L}_k\left(\frac{M^2}{\mathcal{M}_s^2} + \frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)$$

$$T_{\rm NL}^3 = \sum_{i,j,k,i\neq j\neq k}\mathcal{L}_i\left(\frac{2M^2}{\mathcal{M}_s^2}\right)\mathcal{L}_j\left(\frac{M^2}{\mathcal{M}_s^2} + \frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)\mathcal{L}_k\left(\frac{M^2}{\mathcal{M}_s^2} + \frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right),$$

$$T_{\rm NL}^4 = \sum_{i,j,k,i\neq j\neq k}\mathcal{L}_i\left(\frac{M^2}{\mathcal{M}_s^2} + \frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)\mathcal{L}_j\left(\frac{M^2}{\mathcal{M}_s^2} + \frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)\mathcal{L}_k\left(\frac{M^2}{\mathcal{M}_s^2} + \frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right).$$

(62)

Computing the amplitude of 3-point correlation, we obtain

计算三点关联的振幅，我们得到

$$A_{\mathcal{R}} = \sum_{i=1}^{7} B_i S_i \tag{63}$$

where

其中

$$S_1 = 2\mathbf{k}_1 \cdot \mathbf{k}_2 \left[K - \frac{k_1k_2 + k_2k_3 + k_3k_1}{K} - \frac{k_1k_2k_3}{K^2} \right] + \text{ perms },$$

$$S_2 = \frac{2k_1^2k_2^2}{K} + \frac{2k_1^2k_2^2k_3}{K^2} + \text{ perms }, \tag{64}$$

$$S_3 \approx k_3^2 \left[-2K - \frac{2k_1k_2}{K} \right] + \mathcal{C}(z)\, k_3^3 + \text{ perms}$$

$$S_4 = \frac{4k_1^2k_2^2k_3^2}{K^3} + \text{ perms}$$

(65)

$$S_5 = -\frac{K^3}{3} + 2Kk_1k_2 + \frac{k_1k_2k_3}{3} + \text{ perms}$$

$$S_6 = 2(\mathbf{k}_1 \cdot \mathbf{k}_2)\, k_3^2 \left[\frac{2}{K} + \frac{2k_1 + 2k_2}{K^2} + \frac{4k_1k_2}{K^3} \right] + \text{ perms}$$

$$S_7 = (\mathbf{k}_2 \cdot \mathbf{k}_3)\, k_1^2k_3^2 \left[-\frac{2}{K^3} - \frac{6k_2}{K^4} \right],$$

where $z = \frac{K}{K_*}$ with $K_* = a_* H_* = 0.05\text{Mpc}^{-1}$ is a particular reference scale and $C(z) \approx \gamma_E + \ln z - \frac{z^2}{4} + \frac{z^4}{96}$. The derivation of (60) involves using on shell relations for the perturbed mode, i.e., curvature perturbation and the background eigenvalue equation (16) [19, 33] which imply the following key equations:

其中 $z = \frac{K}{K_*}$ 与 $K_* = a_* H_* = 0.05\text{Mpc}^{-1}$ 是特定参考尺度，且 $C(z) \approx \gamma_E + \ln z - \frac{z^2}{4} + \frac{z^4}{96}$ 。式 (60) 的推导用到了扰动模式的在壳关系，即曲率扰动和背景本征值方程 (16) [19, 33]，它们给出以下关键方程:

$$\overline{\Box}_s \mathcal{R} \approx \frac{M^2}{\mathcal{M}_s^2} \mathcal{R} \Rightarrow\ \mathcal{O}\left(\overline{\Box}_s\right) \mathcal{R} \approx \mathcal{O}\left(\frac{M^2}{\mathcal{M}_s^2}\right) \mathcal{R}$$

$$\overline{\Box}_s \mathcal{R}' \approx \left(\frac{\overline{\Box}_s}{\mathcal{M}_s^2} + \frac{\bar{R}_{\text{dS}}}{4\mathcal{M}_s^2}\right) \mathcal{R}' \Rightarrow\ \mathcal{O}\left(\overline{\Box}_s\right) \mathcal{R}' \approx \mathcal{O}\left(\frac{M^2}{\mathcal{M}_s^2} + \frac{\bar{R}_{\text{dS}}}{4\mathcal{M}_s^2}\right) \mathcal{R}',$$

(66)where $\mathcal{O}$ is an arbitrary analytic operator. In the above we used the following commutation relation in the quasi-dS approximation [35]:

其中 $\mathcal{O}$ 是任意解析算符。在上式中我们利用了准德西特近似下的如下对易关系 [35]:

(67)

$$\nabla_\mu \Box_s \phi = \Box_s \nabla_\mu \phi - \frac{R_{\mu\upsilon}}{\mathcal{M}_s^2}\nabla^\upsilon \phi$$

$$\Rightarrow \nabla_\mu F\left(\Box_s\right)\phi \approx \mathcal{F}\left(\Box_s - \frac{R}{4\mathcal{M}_s^2}\right)\nabla_\mu \phi,$$

where ϕ is a scalar. Using (66) we can bring the third-order action of the nonlocal gravity (31) into the local form, and all the effect of non-localities is transferred into the on-shell vertex factors (61) determined by $\mathcal{T}_{\rm NL}$ and $T^1_{\rm NL}, \cdots, T^4_{\rm NL}$. From (60) and (61) we can make a crucial observation that if $\bar{R}_{\rm dS} \gtrsim \mathcal{M}_s^2$, we can expect the exponential of entire function terms in (62) can dominate over the slow-roll suppression, and eventually we get large PNGs, i.e., $f_{NL} \sim \mathcal{O}(1)$. We can see this more clearly by computing the popular limits of f_{NL} called squeezed $f^{\rm sq}_{NL}\left(k_1 \rightarrow \quad 0, k_2 = k_3 = \frac{k}{2}\right)$, equilateral $f^{\rm equiv}_{NL}$ $(k_1 = k_2 = k_3 = k)$, and orthogonal $f^{\rm orth}_{NL}$ $(k_1 = k_2 = k/4, k_3 = k/2)$, which are the useful quantities that are standard targets for CMB observations [100]. Computing $f_{\rm NL}$ for these three limits, we obtain

其中 ϕ 是标量。利用 (66) 我们可将非局域引力 (31) 的三阶作用量化为局域形式，非局域性的全部效应都转移到了由 $\mathcal{T}_{\rm NL}$ 和 $T^1_{\rm NL}, \cdots, T^4_{\rm NL}$ 确定的在壳顶点因子 (61) 中。从 (60) 和 (61) 我们可以得到一个关键结论: 若 $\bar{R}_{\rm dS} \gtrsim \mathcal{M}_s^2$ ， 我们可以预期 (62) 中整函数项的指数会主导慢滚压制，最终得到大原初非高斯性，即 $f_{NL} \sim \mathcal{O}(1)$ 。通过计算 f_{NL} 的常用极限——压缩极限 $f^{\rm sq}_{NL}\left(k_1 \rightarrow \quad 0, k_2 = k_3 = \frac{k}{2}\right)$ 、等边极限 $f^{\rm equiv}_{NL}$ $(k_1 = k_2 = k_3 = k)$ 和正交极限 $f^{\rm orth}_{NL}$ $(k_1 = k_2 = k/4, k_3 = k/2)$ ，我们可以更清楚地看到这一点，这些极限是 CMB 观测标准观测目标中常用的物理量 [100]。对这三种极限计算 $f_{\rm NL}$ ，我们得到

$$f^{\rm sq}_{\rm NL} \approx \frac{5}{12}\left(1-n_s\right) - 35.5\mathcal{T}_{\rm NL} + 8.9\mathcal{C}\left(z\right)\mathcal{T}_{\rm NL} - 1.1\varepsilon^3\frac{\bar{R}^2_{\rm dS}}{\mathcal{M}_s^4}\gamma_S^\dagger\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)$$

$$-\lambda_c\frac{\bar{R}_{\rm dS}}{\mathcal{M}_s^2}\left(5.8\varepsilon^2 T^2_{\rm NL} - 1.5\mathcal{C}\left(z\right)\varepsilon^2 T^2_{\rm NL} - 0.19\varepsilon^3 T^3_{\rm NL}\right),$$

$$f^{\rm equiv}_{\rm NL} \approx \frac{5}{12}\left(1-n_s\right) - 46.6\mathcal{T}_{\rm NL} + 8.9\mathcal{C}\left(z\right)\mathcal{T}_{\rm NL} - 1.8\varepsilon^3\frac{\bar{R}^2_{\rm dS}}{\mathcal{M}_s^4}\gamma_S^\dagger\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)$$

$$-\lambda_c\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\left(7.7\varepsilon^2 T^2_{\rm NL} - 1.5\mathcal{C}\left(z\right)\varepsilon^2 T^2_{\rm NL} - 0.3\varepsilon^3 T^3_{\rm NL} - 0.02\varepsilon^4 T^4_{\rm NL}\right).$$

$$f^{\rm ortho}_{\rm NL} \approx \frac{5}{12}\left(1-n_s\right) - 39.1\mathcal{T}_{\rm NL} + 8.9\mathcal{C}\left(z\right)\mathcal{T}_{\rm NL} - 1.2\varepsilon^3\frac{\bar{R}^2_{\rm dS}}{\mathcal{M}_s^4}\gamma_S^\dagger\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)$$

$$-\lambda_c\frac{\bar{R}_{\rm dS}}{\mathcal{M}_s^2}\left(6.4\varepsilon^2 T^2_{\rm NL} - 1.5\mathcal{C}\left(z\right)\varepsilon^2 T^2_{\rm NL} - 0.2\varepsilon^3 T^3_{\rm NL} - 0.01\varepsilon^4 T^4_{\rm NL}\right).$$

(68)

From (68) we can witness the nonlocal corrections to the $f_{\rm NL}$ in the squeezed, equilateral, and orthogonal limits. Further digressing the expressions (68), there are three types of nonlocal contributions we can notice.

从 (68) 我们可以看到，在压缩、等边、正交三种极限下，非局域对 $f_{\rm NL}$ 的修正。进一步分析式 (68)，我们可以发现一共有三类非局域贡献。

1. Contributions containing $e^{\gamma_S\left(\frac{\bar{R}_{\rm dS}}{\mathcal{M}_s^2}\right)}$ come from the term that is quadratic in scalar curvature (55) for the form factor $\mathcal{F}_R\left(\Box_s\right)$ in (32).

1. 包含 $e^{\gamma_S\left(\frac{\bar{R}_{\rm dS}}{\mathcal{M}_s^2}\right)}$ 的贡献来自 (55) 中标量曲率二次项，对应 (32) 中的形状因子 $\mathcal{F}_R(\Box_s)$。

2. Contributions from the cubic nonlocal scalar curvature term (55): These are the vertex factors containing $T^2_{\rm NL} - T^4_{\rm NL}$. With the form factors (37) and with the conditions imposed in (33), we can easily deduce that

2. 来自三次非局域标量曲率项 (55) 的贡献: 这些是包含 $T^2_{\rm NL} - T^4_{\rm NL}$ 的顶点因子。结合形状因子 (37) 和 (33) 施加的条件，我们可以轻易推得

$$T^1_{\rm NL} \ll T^2_{\rm NL} \ll T^3_{\rm NL} \ll T^4_{\rm NL} \tag{69}$$

in the limit of $M^2 \ll \mathcal{M}_s^2$ and $\bar{R}_{\rm dS} \gtrsim \mathcal{M}_s^2$. Note that since $T^1_{\rm NL}$ does not depend on the ratio $\frac{\bar{R}_{\rm dS}}{\mathcal{M}_s^2}$, its contributions are far less than the contributions involving $T^2_{\rm NL} - T^4_{\rm NL}$, especially for the operators $\ell_i(\Box_s)$ of the form in (38).

在 $M^2 \ll \mathcal{M}_s^2$ 和 $\bar{R}_{\rm dS} \gtrsim \mathcal{M}_s^2$ 的极限下。注意到由于 $T^1_{\rm NL}$ 不依赖于比率 $\frac{\bar{R}_{\rm dS}}{\mathcal{M}_s^2}$，其贡献远小于包含 $T^2_{\rm NL} - T^4_{\rm NL}$ 的项的贡献，尤其是对于 (38) 中形式的算符 $\ell_i(\Box_s)$ 而言。

3. In the vertex factor B_2, the term $\mathcal{C}\left(\frac{K}{K_*}\right)$ comes from taking carefully the infrared (IR) limit of integration using the judicious choice $\tau = -\frac{1}{K_*}$ [101, 102]. Usually, this contribution is slow-roll suppressed in the standard single field models of inflation [102-104], but in our case it is modulated by analytic nonlocal contributions (In (68) small local contributions of the order $O(\varepsilon^2)$ are neglected.) when $\bar{R} \gtrsim \mathcal{M}_s^2$.

3. 在顶点因子 B_2 中，项 $\mathcal{C}\left(\frac{K}{K_*}\right)$ 来源于通过审慎选择 $\tau = -\frac{1}{K_*}$ [101, 102] 对积分的红外 (IR) 极限进行严格处理。通常，该贡献在标准单场暴胀模型 [102-104] 中受慢滚压制，但在我们的场景中，当 $\bar{R} \gtrsim \mathcal{M}_s^2$ 时，它受到解析非局域贡献的调制 (在 (68) 中，量级为 $O(\varepsilon^2)$ 的小局域贡献被忽略)。

From (68) the $f_{\rm NL}$ that can be potentially probed with future observations depends on the following set of quantities:

根据 (68) 式，未来观测可能探测到的 $f_{\rm NL}$ 依赖于以下一组物理量:

$$\left\{ e^{\gamma_S\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)}, \gamma_S^\dagger\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right), \lambda_c, e^{\ell_i\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)} \right\}. \tag{70}$$

As we discussed in the previous section, the scale invariance of bispectrum (63) is broken in the nonlocal R^2-like inflation due to the scale dependence of quantities (70) through

正如我们在上一节讨论的，由于非局域 R^2 类暴胀中 (70) 式物理量存在标度依赖性，(63) 式双谱的标度不变性被破坏，这一过程通过

$$\{\bar{R}_{\rm dS}(k),\ \mathcal{C}(k)\}. \tag{71}$$

Due to the presence of exponentials, we can expect that the scale dependence can be significant. To quantify it, we use the running of $f_{\rm NL}$ defined by Chen et al. [103]

由于指数项的存在，我们可以预期标度依赖性会十分显著。为量化该效应，我们使用 Chen 等人在文献 [103] 中定义的 $f_{\rm NL}$ 跑动:

$$n_{\rm NG} \equiv \frac{d\ln f_{\rm NL}}{d\ln k}, \tag{72}$$

which we can evaluate in the various limits such as squeezed, equilateral, and orthogonal. The $n_{\rm NG}$ can be evaluated using the following quantities:

我们可以在压缩、等边、正交等各种极限下计算它。$n_{\rm NG}$ 可通过以下物理量计算得到:

$$\frac{df_{\rm NL}^{\rm sq}}{d\ln k}$$

$$\approx -\frac{dn_s}{d\ln k} + (-35.5 + 8.9\mathcal{C}(z))\frac{\partial\mathcal{T}_{\rm NL}}{\partial N} + \mathcal{C}^\dagger(z)\left(8.9\mathcal{T}_{\rm NL} + \varepsilon^2\lambda_c\frac{\bar{R}_{\rm dS}}{\mathcal{M}_s^2}T_{\rm NL}^2\right)$$

$$-2.2\varepsilon^4\frac{\bar{R}_{\rm dS}^3}{\mathcal{M}_s^6}\left[\gamma_S^\dagger\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right) + \frac{1}{4}\gamma_S^{\dagger 2}\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right) + \frac{1}{4}\gamma_S^{\dagger\dagger}\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)\right]$$

$$-\varepsilon\lambda_c\frac{2\bar{R}_{\rm dS}}{\mathcal{M}_s^2}\left(5.8\varepsilon^2T_{\rm NL}^2 - 1.5\varepsilon^2\mathcal{C}(z)T_{\rm NL}^2 - 0.76\varepsilon^3T_{\rm NL}^3\right)$$

$$+\varepsilon\lambda_c\frac{\bar{R}_{\rm dS}^2}{2\mathcal{M}_s^4}\left(5.8\varepsilon^2\frac{\partial T_{\rm NL}^2}{\partial N} - 1.5\varepsilon^2\mathcal{C}(z)\frac{\partial T_{\rm NL}^2}{\partial N} - 0.19\varepsilon^3\frac{\partial T_{\rm NL}^3}{\partial N}\right),$$

(73)

$$\frac{df_{\rm NL}^{\rm equiv}}{d\ln k}$$

$$\approx -\frac{dn_s}{d\ln k} + (-46.6 + 8.9\mathcal{C}(z))\frac{\partial\mathcal{T}_{\rm NL}}{\partial N} + \mathcal{C}^\dagger(z)\left(8.9\mathcal{T}_{\rm NL} + \varepsilon^2\lambda_c\frac{\bar{R}_{\rm dS}}{\mathcal{M}_s^2}T_{\rm NL}^2\right)$$

$$-3.6\varepsilon^4\frac{\bar{R}_{\rm dS}^3}{\mathcal{M}_s^6}\left[\gamma_S^\dagger\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right) + \frac{1}{4}\gamma_S^{\dagger 2}\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right) + \frac{1}{4}\gamma_S^{\dagger\dagger}\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)\right]$$

$$-\varepsilon\lambda_c\frac{2\bar{R}_{\rm dS}}{\mathcal{M}_s^2}\left(7.7\varepsilon^2T_{\rm NL}^2 - 1.5\varepsilon^2\mathcal{C}(z)T_{\rm NL}^2 - 1.2\varepsilon^3T_{\rm NL}^3 - 0.12\varepsilon^4T_{\rm NL}^4\right)$$

$$+\varepsilon\lambda_c\frac{\bar{R}_{\rm dS}^2}{2\mathcal{M}_s^4}\left(7.7\varepsilon^2\frac{\partial T_{\rm NL}^2}{\partial T_{\rm NL}^2} - 1.5\varepsilon^2\mathcal{C}(z)\frac{\partial T_{\rm NL}^2}{\partial N} - 0.3\varepsilon^3\frac{\partial T_{\rm NL}^3}{\partial N} - 0.02\varepsilon^4\frac{\partial T_{\rm NL}^4}{\partial N}\right),$$

$$\frac{df_{\rm NL}^{\rm orth}}{d\ln k}$$

$$\approx -\frac{dn_s}{d\ln k} + (-39.1 + 8.9\mathcal{C}(z))\frac{\partial\mathcal{T}_{\rm NL}}{\partial N} + \mathcal{C}^\dagger(z)\left(8.9\mathcal{T}_{\rm NL} + \varepsilon^2\lambda_c\frac{\bar{R}_{\rm dS}}{\mathcal{M}_s^2}T_{\rm NL}^2\right)$$

$$-2.4\varepsilon^4\frac{\bar{R}_{\rm dS}^3}{\mathcal{M}_s^6}\left[\gamma_S^\dagger\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right) + \frac{1}{4}\gamma_S^{\dagger 2}\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right) + \frac{1}{4}\gamma_S^{\dagger\dagger}\left(\frac{\bar{R}_{\rm dS}}{4\mathcal{M}_s^2}\right)\right]$$

$$-\varepsilon\lambda_c \frac{2\bar{R}_{\text{dS}}}{\mathcal{M}_s^2}\left(6.4\varepsilon^2 T_{\text{NL}}^2 - 1.5\varepsilon^2 \mathcal{C}(z)\, T_{\text{NL}}^2 - 0.8\varepsilon^3 T_{\text{NL}}^3 - 0.06\varepsilon^4 T_{\text{NL}}^4\right)$$

$$+\varepsilon\lambda_c \frac{\bar{R}_{\text{dS}}^2}{2\mathcal{M}_s^4}\left(6.4\varepsilon^2 \frac{\partial T_{\text{NL}}^2}{\partial N} - 1.5\mathcal{C}(z)\,\varepsilon^2 \frac{\partial T_{\text{NL}}^2}{\partial N} - 0.2\varepsilon^3 \frac{\partial T_{\text{NL}}^3}{\partial N} - 0.01\varepsilon^4 \frac{\partial T_{\text{NL}}^4}{\partial N}\right).$$

(74)

The relations (74) give us the running of PNGs that depend on the higher derivatives of form factors (70). If we can probe the running of PNGs in future CMB observations, we can reconstruct the structure of form factors and ultimately probe the quantum gravity.

关系式 (74) 给出了依赖于形状因子 (70) 高阶导数的原初非高斯性的跑动。如果我们能在未来的宇宙微波背景观测中探测到原初非高斯性的跑动，我们就能重构形状因子的结构，最终实现对量子引力的探测。

In the context of generalized nonlocal R^2 -like inflation, the interesting feature is the correction to the squeezed limit of f_{NL} despite having single degree of freedom, i.e., scalaron and slow-roll regime. This non-trivial effect is called the violation of the Maldacena consistency relation $f_{\text{NL}}^{\text{eq}} = \frac{5}{12}(1-n_s)$, which we shall discuss in more detail in the next section. This is truly a nonlocal effect which has no analogy with local theories. At the same time, nonlocality also affects the equilateral and orthogonal limits of f_{NL} despite having the sound speed of curvature perturbation unity and with the adiabatic (most often called Bunch-Davies) vacuum [94]. In a way, it is worth to point here that the violation of Maldacena consistency relation in GNLQG is by-product of nonlocality and the curved spacetime. An important point to note here is that the f_{NL} (68) and (74) are derivations computed by summing over all the infinite terms with leading order slow-roll approximations at every step. This implies the results indicate the full nonlocal effect rather than any higher derivative. A natural question of course can be that if the large values of f_{NL} can be achieved in a finite higher derivative theories. First of all, as it is well known, finite higher derivative theories lead to Ostrogradski instabilities due to ghost modes and are not the suitable formulations for quantum gravity with renormalizable properties around Minkowski. Suppose we assume either ghosts are heavy or assume special initial conditions for the ghosts by treating them as tachyons or Lee-Wick modes [80], still we cannot generate large f_{NL} in the finite higher derivative gravity theories unless we heavily fine tune the model parameters with unnaturally large numbers. The GNLQG on the other hand is ghost-free around Minkowski and can be super-renormalizable subjected to the arrangement of form factors with nice properties in the ultraviolet regime.

在推广非局部 R^2 型暴胀的背景下，一个值得关注的特征是，即便该理论仅存在单个自由度即标量子，且处于慢滚区域，仍会对 $f_{\rm NL}$ 的压缩极限产生修正。这种非平凡效应被称为马尔达西纳一致性关系破缺 $f_{\rm NL}^{\rm eq} = \frac{5}{12}(1-n_s)$，我们将在下一节展开详细讨论。这是真正的非局部效应，在局域理论中没有对应物。与此同时，虽然曲率扰动的声速为 1，且真空为绝热真空 (通常称为邦奇-戴维斯真空)[94]，非局域性仍会影响 f_{NL} 的等边限与正交限。从某种角度来说，需要指出 GNLQG 中马尔达西纳一致性关系的破缺是非局域性与弯曲时空的副产物。此处需要注意的重点是，$f_{\rm NL}$ (68) 与式 (74) 的推导是在每一步都对所有无穷多项求和，并采用领头阶慢滚近似得到的。这说明所得结果反映的是完整的非局部效应，而非任何高阶导数修正。自然会产生这样一个问题: $f_{\rm NL}$ 的大数值能否在有限高阶导数理论中实现？首先，众所周知，有限高阶导数理论会因鬼模产生奥斯特罗格拉德斯基不稳定性，不适合作为闵氏背景下可重整化量子引力的表述。假设我们要么认为鬼模质量很大，要么通过将鬼视为快子或李-威克模为其设定特殊初态 [80]，我们仍然无法在有限高阶导数引力理论中得到大的 $f_{\rm NL}$，除非用极不自然的大数值对模型参数进行重度精细调节。而 GNLQG 在闵氏背景下无鬼，且当形状因子在紫外区域具有良好性质时，它是超可重整化的。

Therefore, we conclude finally this section with Fig. 4 which depicts the allowed range of $f_{\rm NL}$ equilateral and orthogonal limits which R^2 -like inflation can successfully predict. Figure 4 does not represent the latest bounds on $f_{\rm NL}^{\rm sq}$, but as we discussed GNLQG does lead to the detectable level of PNGs in the squeezed limit of the reduced bispectrum.

因此，我们在本节最后用图 4 作总结，图中给出了 $f_{\rm NL}$ 型暴胀能成功预言的 R^2 等边限与正交限的允许范围。图 4 并未给出 $f_{\rm NL}^{\rm sq}$ 的最新约束，但正如我们讨论过的，GNLQG 确实会在约化双谱的压缩极限产生可探测水平的原初非高斯性。

On Violation of Maldacena Consistency Relation in GNLQG

广义非局域量子引力中马尔达西那相容性关系的破缺

As we discussed in the previous section, one of the prominent features of PNGs in GNLQG is that the Maldacena consistency relation can be violated, and we can get $f_{\rm NL}^{\rm sq} \sim 1$ depending on the nonlocality scale $\mathcal{M}_s$ and the choice of entire function $\gamma_S(\Box_s)$ in (32). In this section, we discuss how the presence of nonlocality can evade the well-known intuitively derived single-field consistency theorem [95, 105, 106] which says

正如我们在前一节讨论的, 广义非局域量子引力中原初非高斯性的一个显著特征是马尔达西那相容性关系可以被破缺，我们得到的 $f_{\rm NL}^{\rm sq} \sim 1$ 依赖于非局域标度 $\mathcal{M}_s$ 与式 (32) 中整函数 $\gamma_S(\Box_s)$ 的选取。本节我们将讨论非局域性的存在如何规避这一著名的、由直观推导得到的单场相容性定理 [95, 105, 106]，该定理指出

$$f_{\rm NL}^{\rm sq} = \frac{5}{12}(1-n_s) \tag{75}$$

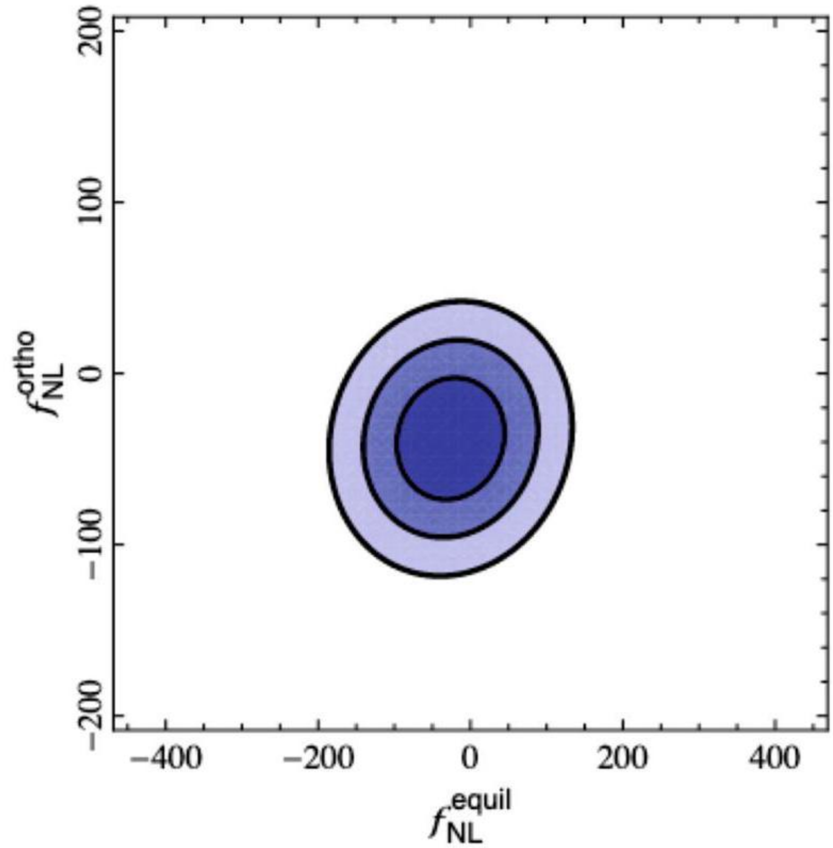
cannot be violated as far as we have single-field, slow-roll, and adiabatic initial conditions. The argument goes with approximating the curvature perturbation as constant on super-Hubble scales. The perturbed metric (say, with wave number k_1) in the so-called unitary gauge in which the inflation perturbations can be put to zero [105-107] can be well estimated to be

只要我们采用单场、慢滚与绝热初始条件，该关系就不可能被破缺。该推导的核心思路是将曲率扰动近似为超哈勃尺度上的常数。在令暴胀扰动为零的所谓幺正规范中 [105-107]，波数为 k_1 的微扰度规可以被估计为

$$ds^2 = -dt^2 + e^{2\zeta_{k_1}} a^2(t)\, dx^2, \tag{76}$$

Fig. 4 In the figure we see 68 %, 95 %, and 99.7 % confidence regions $\left(f_{\rm NL}^{\rm eq}, f_{\rm NL}^{\rm orth}\right) \sim O(10)$ taken from Planck 2018. The predictions of GNLQG R^2 -like inflation lie well within the bounds in this plot along with the squeezed limit $-6 < f_{\rm NL}^{\rm sq} < 4.2$

图 4 图中我们展示了取自普朗克 2018 数据的 68%、95% 和 99.7% 置信区域 $\left(f_{\rm NL}^{\rm eq}, f_{\rm NL}^{\rm orth}\right) \sim O(10)$。广义非局域量子引力 R^2 型暴胀的预测结果与压缩极限 $-6 < f_{\rm NL}^{\rm sq} < 4.2$ 一同很好地落在置信范围内



by neglecting the lapse $\mathcal{N}$ and shift functions $\mathcal{N}_i$ in the well-known Arnowitt-Deser-Misner (ADM) formalism as they are time and spatial derivatives of ζ_{k_1} [95]. Therefore, in the squeezed limit $k_1 \ll k_2 = k_3 = k$ [105, 106] , the 3-point correlation can be Taylor expanded as

我们可以在著名的阿诺维特-德瑟-米斯纳 (ADM) 形式中忽略时移函数 $\mathcal{N}$ 与位移函数 $\mathcal{N}_i$，因为它们都是 ζ_{k_1} 的时间与空间导数 [95]。因此，在压缩极限 $k_1 \ll k_2 = k_3 = k$ [105, 106] 下，三点关联函数可以被泰勒展开为

$$\lim_{k_1\to 0}\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle \approx \lim_{k_1\to 0}\langle\zeta_{\mathbf{k}_1}\langle\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle\rangle$$

$$= -(2\pi)^3\delta^3\left(\sum_i \mathbf{k}_i\right)(n_s-1)\,P_{k_1}P_{k_3}, \tag{77}$$

where P_{k_1}, P_{k_3} are the power spectrum of k_1 mode and k_3 mode, respectively. The above result relies on the trick of rescaling the spatial coordinates $x^i \to e^{\zeta_{k_1}}x^i$ since ζ_{k_1} can be treated as a constant on super-Hubble scales [106-108]. The reason why (77) cannot hold in nonlocal theory lies in the details of approximations performed in this result.

其中 P_{k_1}, P_{k_3} 分别是 k_1 模和 k_3 模的功率谱。上述结果依赖于对空间坐标 $x^i \to e^{\zeta_{k_1}}x^i$ 重新标度的技巧，因为 ζ_{k_1} 可被视为超哈勃尺度上的常数 [106-108]。(77) 式在非局域理论中不成立的根源，在于推导该结果时所用近似的具体细节。

For better illustration, let us consider an action of the following form which is a part of the full action (31):

为了更清楚地说明，我们考虑如下形式的作用量，它是完整作用量 (31) 的一部分:

$$S^{\text{Nonlocal}}_{R+R^2+R^3} = S^{\text{local}}_{R+R^2} + S^{\text{Nonlocal}}_{R^3}, \tag{78}$$

where $S^{\text{local}}_{R+R^2}, S^{\text{Nonlocal}}_{R^3}$ are defined in (55) with condition (33).

其中 $S^{\text{local}}_{R+R^2}, S^{\text{Nonlocal}}_{R^3}$ 由满足条件 (33) 的式 (55) 定义。

The second-order perturbed action of (78) around FLRW background of (16) can be computed as

我们可以计算出 (78) 式在 (16) 式 FLRW 背景附近的二阶微扰作用量为

$$\delta^{(2)}S^{\text{Nonlocal}}_{R+R^2+R^3} = \delta^{(2)}S^{\text{local}}_{R+R^2}, \tag{79}$$

where $\delta^{(2)}S^{\text{Nonlocal}}_{R^3}\big|_{\bar{\Box}\bar{R}=M^2\bar{R}} = 0$, which is the result derived in [32]. This implies the theory is completely local at the second-order perturbation level of the action. This means the approximation of curvature perturbation constant on super-horizon scales perfectly holds here. But obviously in this example, we get nontrivial nonlocal contributions at the third-order perturbed level of the action which is exactly why we get violation of consistency relation. Clearly (78) is a counterexample to the derivation of (77). This happens because in the nonlocal case the relations (66) lead to the enhancement of the interaction strength between the long wavelength mode and short wavelength modes especially when $\bar{R} \gtrsim \mathcal{M}_s^2$.

其中 $\delta^{(2)}S^{\text{Nonlocal}}_{R^3}\big|_{\bar{\Box}\bar{R}=M^2\bar{R}} = 0$，这是文献 [32] 中推导得到的结果。这意味着该理论在作用量的二阶微扰层面是完全局域的，说明曲率扰动在超哈勃尺度上为常数的近似在这里完全成立。但显然在这个例子中，我们在作用量的三阶微扰层面得到了非平凡的非局域贡献，这正是我们得到相容性关系破缺的原因。显然 (78) 式是对 (77) 式推导的一个反例。这种情况发生的原因是，在非局域情形下，(66) 式的关系会增强长波长模式与短波长模式之间的相互作用强度，尤其是当 $\bar{R} \gtrsim \mathcal{M}_s^2$ 时。

So in summary, the proof (77) is ideally valid only in local theories, in particular two derivative theories. In the case of nonlocal theory, one cannot rescale a metric with a local operator and Taylor expand 3-point function because of nonlocality followed by the noncommutativity of d'Alembertian and covariant derivatives in curved spacetime (66).

因此总而言之，证明 (77) 理想情况下仅在定域理论中成立，尤其是二阶导数理论。在非定域理论的情况下，由于非定域性，以及弯曲时空中达朗贝尔算符与协变导数不对易 (66)，我们无法通过定域算符重新标度度规，也无法对三点函数做泰勒展开。

Lessons from Generalized Nonlocal R^2 -like Inflation and Its Impact on the EFTs of Inflationary Cosmology

推广的非局域 R^2 型暴胀的经验及其对暴胀宇宙学有效场论的影响

The R^2 -like inflation we get here in GNLQG totally relies on the principles of building quantum gravity that is totally different from the construction of EFTs in inflationary cosmology (EFTI) where strong assumptions are usually made [44, 45, 109] . Here we discuss briefly the differences one can get in the context of EFTI and GNLQG nonlocal inflation (but see [32,33] for more detailed analysis). First of all, EFT of single inflation (EFT-SI) [44] prescribes that any new physics in the context of inflation has to emerge from the nontrivial sounds speeds of perturbed degrees of freedom. Surely, the context of GNLQG is a counterexample to this because the sound speeds of perturbed modes are unity, but still we get new effects in the inflationary correlations thanks to nonlocality. This is an important point to rethink and expand the meaning of so-called EFTs in cosmology. As a simple example, let us consider the following term in the action that Weinberg has proposed [109] in the context of EFT-SI:

我们在广义非局域量子引力 (GNLQG) 中得到的 R^2 型暴胀完全基于量子引力的构建原则，这与暴胀宇宙学有效场论 (EFTI) 的构建完全不同，后者通常需要做出强假设 [44, 45, 109] 。在此我们简要讨论 EFTI 与 GNLQG 非局域暴胀框架中的差异 (更详细分析见文献 [32,33])。首先，单场暴胀有效场论 (EFT-SI)[44] 指出，暴胀框架中的任何新物理都必然源自扰动自由度的非平凡声速。显然，GNLQG 框架就是该观点的反例: 其扰动模式的声速为 1，但由于非局域性，我们仍能在暴胀关联函数中得到新效应。这是一个重要的切入点，值得我们重新思考并拓展宇宙学中所谓有效场论的内涵。举一个简单例子，我们来看温伯格在 EFT-SI 框架下提出的作用量中的如下项 [109]:

$$\int d^4x\sqrt{-g}\left[f_1\left(\frac{\phi}{\Lambda}\right)W^{\mu\nu\rho\sigma}W_{\mu\nu\rho\sigma}\right], \tag{80}$$

where ϕ is a canonically normalized inflation field or the Goldstone boson in the language of EFT-SI proposed in [44]. The effective scale is $\Lambda < M_p$, and if $\phi > M_p$ that is very much natural to happen for majority of single field models of inflation (the large field ones), for example, in the local R^2 and Higgs inflation [110], then the term (80) may not be the lowest order term. Therefore, a simple generalization of (80) would be the following:

其中 ϕ 是正则归一化的暴胀场，或是 [44] 提出的 EFT-SI 表述中的戈德斯通玻色子。有效标度为 $\Lambda < M_p$ ，若满足 $\phi > M_p$ ——这对大多数单场暴胀模型 (大场模型) 而言非常自然，例如局域 R^2 模型与希格斯暴胀 [110]，那么式 (80) 可能就不是最低阶项。因此对式 (80) 的一个简单推广如下:

$$\int d^4x\sqrt{-g}\left[f_1\left(\frac{\phi}{\Lambda}\right)W^{\mu\nu\rho\sigma}\mathcal{F}_1\left(\frac{\Box}{\Lambda^2},\frac{R}{\Lambda^2}\right)W_{\mu\nu\rho\sigma}\right], \tag{81}$$

where $\mathcal{F}_1\left(\frac{\Box}{\Lambda^2},\frac{R}{\Lambda^2}\right)$ is an analytic nonpolynomial functions of d'Alembertian and the Ricci scalar. We cannot truncate the terms in (81) especially if $R \gtrsim \Lambda^2, \Box \gtrsim \Lambda^2$, and if $\phi \sim O\left(M_p\right)$ during inflation, one can expect $f_1\left(\frac{\phi}{\Lambda}\right) \gg 1$. We can easily notice a similarity of (81) with the third term in the first line of GNLQG (31). Suppose if we only consider the lowest order term $\frac{R}{\mathcal{M}_s^2}W_{\mu\nu\upsilon\sigma}W^{\mu\upsilon\rho\sigma}$, we can verify that we would end up with a ghost and also a nontrivial sound speed for tensor mode [111]. A catch here is that the sound speed of perturbed modes could just be an artifact of a bad truncation of the fundamental theory rather than a feature that can persist at UV completion. Obviously there could be fundamental theories with nontrivial sound speeds all the way up to UV scales [112]. The EFTs of multifield inflation [45] are known to produce different PNGs especially in the squeezed limit. However, a consistent inflationary solution in multifield inflation requires the slow-roll parameters (including the sound speeds) to be slowly varying. This would render the signature for running PNGs to be small, but in the context of PNGs in GNLQG it was found that the running of PNGs can be at least of the order of magnitude higher than those EFT models. If this is found in the future observations [51-55], we would definitely get a lead on quantum gravity research. Similarly, the predictions of running of tensor tilt in the nonlocal R^2 -like model can be an order of magnitude higher than in the EFT frameworks of inflation [3].

其中 $\mathcal{F}_1\left(\frac{\Box}{\Lambda^2},\frac{R}{\Lambda^2}\right)$ 是达朗贝尔算符与里奇标量的解析非多项式函数。我们不能截断式 (81) 中的项，尤其是当 $R \gtrsim \Lambda^2, \Box \gtrsim \Lambda^2$ 时；若暴胀过程中满足 $\phi \sim O\left(M_p\right)$ ，则可以预期得到 $f_1\left(\frac{\phi}{\Lambda}\right) \gg 1$ 。我们很容易发现式 (81) 与 GNLQG 式 (31) 第一行中的第三项存在相似性。假设我们仅考虑最低阶项 $\frac{R}{\mathcal{M}_s^2}W_{\mu\nu\upsilon\sigma}W^{\mu\upsilon\rho\sigma}$ ，可以验证我们最终会得到一个鬼场，以及张量模式非平凡的声速 [111]。这里需要注意，扰动模式的声速可能只是基础理论不当截断带来的人为结果，而非在紫外完备化后依然存在的物理特征。显然，也存在一些基础理论，其非平凡声速可以一直保持到紫外能标 [112]。据我们所知，多场暴胀有效场论 [45] 会产生不同的原初非高斯性，尤其是在压缩极限下。然而，多场暴胀中自洽的暴胀解要求慢滚参数 (包括声速) 满足慢变条件，这会使得原初非高斯性跑动的信号很小，但在 GNLQG 的原初非高斯性框架中，我们发现原初非高斯性跑动的幅度至少比上述有效场论模型高一个数量级。如果未来观测 [51-55] 探测到这一结果，我们无疑会得到量子引力研究的重要线索。类似地，非局域 R^2 类模型中张量倾角跑动的预言，也比暴胀有效场论框架的结果高一个数量级 [3]。

In a nutshell, developments of GNLQG do leave an impact on quantum gravity research, and it highlights the importance of higher curvature terms and nonlocality in building a ghost-free quantum gravity model in curved spacetime and embedding the famous R^2 model of inflation into it. On the other hand, GNLQG sheds light on inflationary cosmological observables. Being precise, GNLQG not only presents a good counterexample to the so-called EFT of inflation but also pushes the future observations to look for running of PNGs and running of tensor tilt in order to precisely probe early Universe physics.

简而言之, 广义非局部二次引力 (GNLQG) 的发展确实对量子引力研究产生了影响, 它凸显了高阶曲率项与非局域性在构建弯曲时空中无鬼场的量子引力模型，并将著名的 R^2 暴胀模型嵌入该框架过程中的重要性。另一方面, GNLQG 也为暴胀宇宙学可观测效应研究提供了启示。准确来说, GNLQG 不仅给所谓的暴胀有效场论提供了一个很好的反例, 还推动未来观测去寻找原初非高斯性 (PNG) 的跑动和张量倾角的跑动，以精准探测早期宇宙物理。

Conclusion and Outlook

结论与展望

It is very hard to separate cosmology and quantum gravity research as developments in each field imply understandings in the other. In this chapter, we have elaborated this in the context of R^2 inflation and its connection toward achieving a consistent quantum gravity. If one keeps the spirit of R^2 inflation and the success of Stelle gravity as a renormalizable theory, then as we have demonstrated in this chapter, a rather unavoidable route toward UV completion is presented by nonlocal higher curvature gravitational theories. We have reviewed earlier developments of the nonlocal quadratic curvature gravity (NLQG) theories which are very much possible candidates to build a consistent quantum gravity theory. We have clearly pointed out that NLQG formulation is only consistent in the flat Minkowski spacetime, so in the context of (early Universe) cosmology NLQG must be extended with higher curvature nonlocal terms. This deeply elucidates why we must in parallel aim to go forward in quantum gravity research with cosmological application. We have reviewed the recent construction of the most general theory of nonlocal gravity (up to cubic terms in curvature) that admits R^2 -like inflation. The generalized nonlocal quantum gravity (GNLQG) theory can be formulated with a finite parameter space by demanding ghost-freeness in the inflationary epoch. Furthermore, with appropriate assumptions on the form factors, the presence of higher curvature nonlocal terms can preserve the super-renormalizable property that was understood NLQG. This renders GNLQG as indeed a very much promising way forward where more details can be found in [32].

宇宙学与量子引力研究很难分割开，因为两个领域各自的发展都会推动对另一领域的理解。本章我们围绕 R^2 型暴胀及其与构建自洽量子引力的关联详细阐述了这一点。如果秉持 R^2 型暴胀的核心思想，同时承认施泰勒引力作为可重整化理论的成功之处，那么正如本章所证明的，紫外完备的一条几乎不可避免的路径就是由非局域高阶曲率引力理论给出的。我们回顾了非局域二次曲率引力 (NLQG) 的早期发展，该理论完全有资格成为构建自洽量子引力理论的候选者。我们明确指出，NLQG 的表述仅在平直闵氏时空下自洽，因此在 (早期宇宙) 宇宙学的语境下，必须为 NLQG 引入额外的高阶曲率非局域项。这清楚地说明，我们在推进量子引力研究的同时，必须兼顾其宇宙学应用。我们回顾了近期构造的最广义非局域引力理论 (曲率项保留至三阶)，该理论允许存在 R^2 型暴胀。通过要求暴胀时期无鬼，广义非局域量子引力 (GNLQG) 理论可以在有限参数空间内表述。此外，对形状因子做出合理假设后，新增的高阶曲率非局域项可以保留 NLQG 原有的超可重整化性质。这使得 GNLQG 成为量子引力研究非常有前景的发展方向，更多细节可参见文献 [32]。

We have further elaborated in this chapter that R^2 -like inflation in GNLQG is predictive and thus can be probed by future CMB, PGW, and Large-Scale Structure observations [47,48,50-56]. The takeaway message here is that GNLQG gives a new view on expected outcomes in the framework of inflation and significantly impacts our predictions of possible correlations in the present observational data as compared to the so far well-explored EFTs of inflationary cosmology. Surely, there is a long way to go to understand quantum gravity. However, the content of this chapter provides new ways for investigation of GNLQG in curved spacetimes other than inflationary ones, such as black holes and anisotropic cosmology. It is also important to further understand more cosmological correlations in GNLQG involving gravitons which are under current investigation. Furthermore, GNLQG opens new doors for quantum gravity research beyond NLQG.

> 我们在本章中进一步阐明，GNLQG 中的 R^2 型暴胀具有可预言性，因此可以被未来的宇宙微波背景 (CMB)、原初引力波 (PGW) 以及大尺度结构观测检验 [47,48,50-56]。本文核心结论是，GNLQG 为暴胀框架下的预期结果提供了新视角，和目前研究已经非常充分的暴胀宇宙学有效场论相比，它对现有观测数据中可能存在的关联给出了截然不同的预言。诚然，理解量子引力还有很长的路要走，但本章内容为 GNLQG 在除暴胀背景之外的弯曲时空 (比如黑洞和各向异性宇宙学) 中的研究提供了新思路。进一步理解 GNLQG 中包含引力子的宇宙学关联也十分重要，相关研究目前正在推进。此外，GNLQG 也为超出 NLQG 范围的量子引力研究打开了新的大门。

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